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Statistical Efficiency

THE STATISTICAL EFFICIENCY OF ECHO SURVEYS WITH SPECIAL REFERENCE TO LAKE TANGANYIKA

by

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PREPARATION OF THIS DOCUMENT

As science is becoming more and more important in African inland waters where FAO Projects are operating, new techniques have been used in a variety of instances for quick assessment of the characteristics of the surveyed populations. A quantitative acoustic survey carried out in Lake Tanganyika, in cooperation with the Fishery Projects of Tanzania and Burundi, aims to map the distribution and estimate the abundance of the pelagic fish in the lake. This document discusses the statistical aspect of the echo survey design and operation which govern the accuracy and precision of the abundance estimation.

Acknowledgement

I would like to thank Messrs. D. Chapman, M. Mann, O. Enderlein, W. Ferro and especially R.A. Ryder and K.A. Johannesson, who have provided helpful criticism.

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ABSTRACT

An acoustic survey[1/] assessed the total biomass and distribution of pelagic fish in Lake Tanganyika (Lat. 5°S, Long. 29°E) in November 1973. The lake was post-stratified and by using graphic methods, total biomass of the surveyed population was estimated at about 2.8 million metric tons. Night sample observations were significantly higher than day-light observations with an estimated average ratio 4:1. Night estimates were strongly over-dispersed and fitted the negative binomial distribution. Day estimates were also over-dispersed and after a proper smoothing of the empirical data fitted the negative binomial distribution. The arithmetic data were log-normalized for use in any further statistical analysis.

The acoustic surveys have been compared with the traffic surveys and suggestions have been made for the development of Quality Check Surveys of Echo Surveys.

The Mean Difference of GINI has proved to be useful for providing indications of the level of dispersion of pelagic fish in the lake by stratum and by day and night periods. Also, the dispersion pattern of pelagic fish in the lake can effectively be portrayed by use of oblique projection charts.

The sources and causes of potential errors inherent in echo surveys are discussed and their spectrum has been classified into domains by taking into account the nature of the errors. It is indicated that the joint effect of the various sources of errors (sampling and non-sampling errors) are responsible for the observed differences between night and day sample observations.

An estimator based on the method of collapsed strata has proved effective for estimating both the size of the total biomass of pelagic fish and its precision. It has been proved that the level of efficiency of the line sample of the survey is a function of the density of fish, and that for increasing the precision of sample estimates the distance between tracks should be reduced in proportion to the density of the surveyed fish population.

1/ The required data for this study were kindly provided by Mr. K.A. Johannesson, Fishery Resources Officer (Acoustic Surveys), FAO; see also: K.A. Johannesson, Preliminary quantitative estimates of pelagic fish stocks in Lake Tanganyika by use of echo integration methods, EIFAC/74/I/Symp.54

CONTENTS

CHAPTER 1: AN OUTLINE OF THE PROBLEM

1.1 Introduction

In the field of applied fishery research a quick assessment of the absolute abundance and distribution of fish stocks is of a great theoretical and practical importance. Acoustic assessment techniques have been used in a variety of instances and are rapidly replacing the older classical methods, primarily because of the rapidity with which estimates can be made and the relatively low cost of operation beyond the initial outlay.

1.2 The nature of the problem

The large-scale acoustic survey at Lake Tanganyika (surface area of 10 000 n mi^2) was apparently the largest of its kind in freshwater research, a fact that increases the importance of the obtained results and their interpretation. A critical evaluation of the survey system of the survey would provide the information needed for an assessment of the level of efficiency of the design of the survey and the level of reliability of the sample estimates. Specifically, this paper services two purposes: (a) According to the design of the survey, estimates of the absolute biomass were calculated by using a "graphical" method. This method of estimation, although simple, cannot be used to calculate the precision of the sample estimates. The need for developing proper estimators for the survey is obvious; (b) Echo surveys should be classified in the category of "sophisticated" surveys. Although the quality of the field operations of the echo survey in Lake Tanganyika was high it is very desirable at this stage to check the validity of the measuring device. Specifically, by taking into account the peculiarities of the surveyed population, we detect the potential sources of errors inherent in the survey system of the survey and estimate their effect on the results of the survey.

CHAPTER 2: THE DISTRIBUTION OF FLOW OF PELAGIC FISH

By using imagination one can compare the flow of fish at any point in a lake per unit of time with traffic flow. The specimen-mileage corresponds to vehicle-mileage and expresses the number of miles swum by a fish in a water-road (three-dimensional space) or set of roads during a specified period. In order to simplify my analytical work I would like to supplement my terminology describing the distribution of flow of pelagic fish:

(a) Total survey area (A): Is the total area in a lake covered by an echo survey.

(b) If one can imagine that, at time t_o, all the water-roads laid down end to end, the corresponding total area (surface area) would be called the total fish area1/ (A_f).

(c) The general "packing"2/ of fish within a water-road or set of water-roads can be expressed by layers of fish or shoals of fish. Further, a scaling system should be established expressing qualitatively the level of packing of fish, e.g. very dense, dense, etc. A quantitative assessment of the level of packing of fish is expressed by the quantity of fish per area unit (number of fish/area unit or weight of fish/area unit).

1/ Although a volume-unit is more realistic than an area-unit in this context, to simplify our analysis one can think of fish in a three-dimensional sense in a lake being projected on a two-dimensional surface

2/ From an aggregation point of view

It seems to me that, in any attempt for estimating the dispersion pattern of a fish population at time t_o, the peculiarities of the survey population should be taken into account. One can see specific dispersion patterns rather than a general dispersion pattern of the fish population. For example, the general packing of one sub-population of fish, the quantitative level of packing of fish, and the spatial distribution of fish in a set of roads might be different from that of a second sub-population of fish occupying another set of roads. Innate behaviour patterns of individuals within a shoal and other factors might be responsible for the statistical significance between the organic structure of the sub-populations.

As one can see, our imaginery water-road system is not fixed over time. Also, on a static basis, it is impossible to use simple geographical rules in order to define a water-road system in a lake. If this information was available at the process of designing an echo survey, the efficiency of the sampling design of the survey would be very high.

It has been argued that an objective assessment of the distribution of flow of pelagic fish is impossible prior to the design of an echo survey. However, indications of the distribution of flow of pelagic fish at time t_o, can be obtained after the survey has been conducted and by using the sample observations of a properly designed echo survey.

CHAPTER 3: SAMPLE SURVEY PROCEDURE (LAKE TANGANYIKA)

3.1 Composition of pelagic stock1/

In Lake Tanganyika all pelagic biomass is composed of six species. The prey include the two small clupeids Limnothrissa miodon and Stolothrissa tanganyika. The four predators comprising three very similar endemic species of the genus Lates (L. angustiphrons, L. mariae and L. microlepis), and another endemic predator of smaller size and presumably shorter life history, Luciolates stappersii. Catch data indicate that more than half of the catch is composed of clupeids. In the survey reported here no attempt was made to differentiate biomass estimates by species.

3.2 The sampling method

From a statistical point of view the sampling method of the echo survey at Lake Tanganyika can be described as a Stratified Random Line Sample (SRLS). Past experience has proved that this method of sampling is useful when an area sampling frame is used for the selection of the sample of a survey2/. In our case, predetermined survey tracks were designed as a parallel grid with a total of 48 transects across practically the whole length of the lake at 6 n mi intervals for the northern half of the lake, and at 10 n mi intervals for the southern half. The survey track in Burundi/Zaire waters was run twice giving two estimates for that part of the lake which were separated by a 14-day interval3/.

1/ See also: Henderson, Ryder and Kudhongania (1973): Assessing Fisheries Potentials of Lakes and Reservoirs, FAO-FI:FMD/73/S-32

2/ See also: The Design of Fisheries Statistical Surveys, Inland Waters, FAO-FIPS/T.133, by G.P. Bazigos, p.80

3/ The field work of the survey was carried out during a 30-day period between 20 October abd 24 November 1973, of which 12 days were spent working out methods to catch and preserve live fish, in order to calibrate the equipment experimentally against pelagic fish species

3.3 Field operations[1/]

For the field operations of the survey the research vessel LADY ALICE II of the Burundi Fishery Project (LOA 39 ft) was used. A Simrad scientific sounding system, consisting of an EK 120 (120 KHZ) sounder coupled with complementary electronic instrumentation, was temporarily installed in the wheelhouse. The instruments included: echo integrator QM-MK II, Hewlett Packard storage oscilloscope, electronic counter, signal generator, AC amplifier-voltmeter, and signal attenuator. A calibrated test hydrophone was also included.

For survey purposes the vessel speed was adjusted to 6 knots and continuous manual data logging was carried out every 20 minutes, thus giving 2 n mi as the elemental sampling distance unit (ESDU).

Generally, the two integrator channels (A and B), with 10 decibel difference in gain setting, were adjusted to cover the whole depth interval of the fish distribution. This procedure prevented vertical stratification of the echo integration, but it made it possible to cope with the wide dynamic range of echo intensities encountered.

3.4 The sample of the survey

In order to simplify the field operations of the survey and provide estimates on a regional basis, the lake was divided into five areas, here called strata. In the Table below (Table 3.4.1) the established strata are given. Also, information is provided on the number of sample ESDUs and area of individual stratum (n mi^2) of the surveyed population.

Table 3.4.1 The sample of the survey

Strata		No. of sample ESDUs		Total survey area	
Name	Location	2 mi	%	n mi^2	%
TOTAL:	(Lat.)	600	100.00	6 458	100.00
1. Burundi/ Cap Banza	03°20'- 04°30'S	104	17.33	698	10.81
2. Area near Kigoma	04°30'- 05°40'S	175	29.17	1 410	21.83
3. Area near Lagosa	05°40'- 06°30'S	104	17.33	1 121	17.36
4. Area near Kipili/Karema	06°30'- 08°00'S	160	26.67	2 446	37.88
5. Zambia	08°00'- 08°45'S	57	9.50	783	12.12

From the tabulated data (Table 3.4.1) one can see that the allocation of the total number of sample ESDUs to the strata is more or less proportional to the area of the strata.

1/ See footnote on page iv

3.5 The procedure for estimating total biomass

According to the sampling design of the survey the estimated total fish biomass ($\hat{W}_\beta$, weight) can be regarded as the product of two magnitudes[1/]:

(1) $$\hat{W}_\beta = \sum_{h=1}^{5} A_h \hat{\rho}_{h(\beta)}$$

where,

A_h : Area of the survey population (n mi²) in the h^{th} stratum

$\hat{\rho}_{h(\beta)}$: Estimated average fish biomass density (t) per area sampling unit (ASU = 1 n mi²) in the h^{th} stratum

Further, $\hat{\rho}_{h(\beta)}$ is the product of two magnitudes each of which is estimated independently.

(2) $$\hat{\rho}_{h(\beta)} = \hat{M}_h \hat{C}$$

where,

$\hat{M}_h$: Estimated average integrator readings (mm) per ESDU in the h^{th} stratum

$\hat{C}$: Estimated general converting factor. On the basis of $\hat{C}$ the integrator readings per elemental sampling distance unit are converted into metric tons per ASU (t/n mi²).

For the estimation of $\hat{M}$ the sample data of the acoustic survey were used. An estimate of C was obtained through an ad hoc simulation survey, i.e. calibration of the acoustic equipment experimentally against a standard density of live fish (see footnote on page iv).

From the foregoing discussion it is obvious that the level of reliability of $\hat{W}_\beta$ is a function of the level of reliability of $\hat{\rho}_\beta$, which in turn is a function of the levels of reliability of $\hat{C}$ and $\hat{M}$.

CHAPTER 4: A QUICK VALIDATION OF THE RESULTS OF ECHO SURVEYS

4.1 Introduction

In this chapter, the need for development of quality check surveys for a quick validation of the results of main echo surveys is discussed.

4.2 Quality Check Surveys of Echo Surveys (QCS-ES)

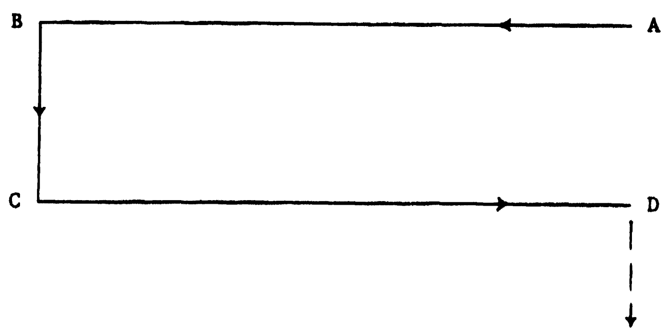
It has been discussed in a previous chapter (Chapter 2) that by using imagination the flow of fish at any point in a lake per unit of time can be compared with traffic flow. In such a case, one can argue that sampling methods can be developed to provide reliable estimates both of the volume of fish traffic on a static basis and the trend of this traffic on a dynamic basis. Further, the results of a properly designed fish traffic survey, based on small samples and running concurrently with an echo survey, can be used to check the level of reliability of the results of the main echo survey. For our purposes we shall call the specific fish traffic survey Quality Check Survey of the Echo Survey (QCS-ES).

1/ For integration purposes, in this paper we can use the same notation as employed by K.A. Johannesson; see footnote on page iv

It should be noted that the object of the echo survey at Lake Tanganyika was to provide quick estimates of the area distribution and absolute abundance of the pelagic fish biomass in the lake. For this purpose the sampling method of the survey was based on Stratified Random Line Sample (see Chapter 3). By using imagination one can see that the sample tracks of the survey are check points (lines) in the water-roads, and the sample observations indicate the volume of fish passing through the check points at instantaneous periods of time. As one can observe, the echo survey does not collect information on the volume of fish traffic.

The justification of my theoretical proposals for using specific fish traffic surveys for a quick validation of the results of echo surveys can be explained in simple language as follows. Assuming that by using certain control characteristics for stratification the lake is divided into a number of vertical hydrographic zones and that each zone represents a set of water-roads. Assuming also that a systematic sample of check points is selected within each zone, then by using a proper device, information can be collected at each sample check point on the fish traffic for specified periods of time. The length of the "reference period" at checking points should be a function of the dispersion pattern and the level of movement of pelagic fish. As a parenthesis, I would like to say here that, in my opinion, one of the sources of errors responsible for the observed differences between night and day measurements could be attributed to the experimental conditions of the survey as far as the "reference period" is concerned[1/]. Specifically, in the echo survey the sample observations were based on instantaneous reference periods of time (t_ε) for day and night measurements. By doing so we introduced the assumption of homogeneity over time as far as dispersion pattern and level of movements of pelagic fish are concerned. This, however, is not the case in Lake Tanganyika.

In the Figure below a grid of the line sample of the survey is portrayed, consisting of two parallel tracks (AB, CD) and the vertical line BC.



From the foregoing discussion one can note that only the parallel lines AB, CD should be considered as check points in our water-road system. Measurements along the line BC should be excluded because they provide information on the following two heterogeneous magnitudes: (a) vertical distribution and abundance of pelagic fish moving in the same direction with the research vessel; and (b) measurements of the volume of fish crossing the line on an instantaneous reference period basis.

If there was regularity in the dispersion pattern and the level of movement of pelagic fish, one could expect that the sample observations at AB should not differ significantly from those at CD, and that the t_ε should be considered as the proper reference period for the survey system of the survey. In such a case the need of a QCS-ES can hardly be justified. One can argue that differences in the values of average integrator readings per elemental sampling distance unit between the parallel lines (total or stratified), give an indication of the level of irregularity in the dispersion pattern and level of movements of pelagic fish[2/].

1/ See Chapter 6

2/ See Chapter 5

QCS-ESs are of value when the assumption of regularity as described above is not valid. In such a case QCSs are needed to detect the potential sources of error and to estimate their effect on the results of the main echo surveys.

External estimates, i.e. estimates obtained through classical methods of estimating standing crops of fish, can also be used in order to provide indications of the level of validity of the results of an echo survey.

CHAPTER 5: ESTIMATED DISPERSION PATTERN OF PELAGIC FISH IN LAKE TANGANYIKA

5.1 Introduction

In this Chapter, by using the sample data of the echo survey, indications are provided regarding the dispersion pattern of pelagic fish in the lake. To permit a simple presentation of the dispersion pattern, elementary statistical methods have been used[1/].

5.2 Estimated fish area (A_f)

Are pelagic fish everywhere in the lake (survey area)[2/]? The answer to the question is no. The estimated confidence limits for the proportion of ESDUs with fish (p_i) are for the lake as a whole 69 percent lower limit, and 80 percent upper limit ($\hat{a}$ = 5 percent). It is worth noting that, with the exception of Str.1, there are no significant differences in the estimated values of p_i between the individual stratum and for day and night measurements within strata. The Table below (Table 5.2.1) gives the estimated relative distribution of the survey area into fish area (p_i), and the remaining area ($q_i = 1-p_i$) by stratum and by day and night periods of time. Also, Table 5.2.2 gives the estimated confidence intervals for $\hat{P}_i$ ($a = 0.05$).

Table 5.2.1 Estimated percentage distribution of the survey area into fish area (p_i) and the remaining area (q_i) by stratum and by day and night periods of time

Space	Proportion	Total	Str.1	Str.2	Str.3	Str.4	Str.5
Day time/proportions							
Total:	-	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Fish area	p_i	0.7457	0.7500	0.8072	0.6591	0.7812	0.3333
Remaining area	q_i	0.2543	0.2500	0.1928	0.3409	0.2188	0.6667
Night time/proportions							
Total:	-	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Fish area	p_i	0.6973	0.9200	0.6167	0.8000	0.7500	0.4167
Remaining area	q_i	0.3027	0.0800	0.3833	0.2000	0.2500	0.5833

1/ See also Allendix I, Mathematical Model for Acoustic Survey in Lake Tanganyika

2/ For our analysis the sample data of the horizontal tracks only were used

Table 5.2.2 Estimated confidence limits for P_i
(P = 95%)

Day time											
Total		Str.1		Str.2		Str.3		Str.4		Str.5	
Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.
0.6895	0.8019	0.5976*	0.8024*	0.7218	0.8926	0.5174	0.8008	0.6791	0.8833	0	0.7176
Night time											
0.6307	0.7639	0.8030*	1.0000*	0.4926	0.7408	0.6544	0.9456	0.6206	0.8794	0.2136	0.6198

* The samples have been selected from two different populations as far as the surveyed characteristic is concerned

Figure 5.2.1 provides a graphical explanation of the relative composition of the survey area. As one can see, an estimate of the absolute size of the fish area (A_f) can be calculated by applying the estimated proportions (p_i) on the total survey area which is a known magnitude (see Chapter 7).

5.3 Estimated level of dispersion

In our case the Mean Difference of GINI[1/] (d_1) can be used in order to provide indications of the level of dispersion of pelagic fish in the lake. Specifically, GINI's method can be employed to provide both estimates of the absolute differences between one unit and another of the survey variate and a measure of the level of dispersion of the respective empirical distributions(average of the absolute differences between one unit and another).

By using the sample data I prepared the respective matrices indicating the absolute differences between the individual values of ESDUs (integrator's readings, mm - see Appendix II). Specifically, calculations were made on a stratum basis and separately for day and night periods of time (0600-2000, 2000-0600).

In the Table below (Table 5.3.1) the estimated values of d_1 are given by stratum and by day and night periods[2/].

Table 5.3.1 Estimated values of d_1 and average integrator readings per ESDU ($\bar{y}$ = $\hat{M}$) by stratum and by day and night periods

Space	$\hat{d}_1$ values (mm)		$\hat{R}_1 = \frac{(2)}{(1)}$	$\bar{y}(=\hat{M})$		$\hat{R}_2 = \frac{(5)}{(4)}$
	Day (1)	Night (2)	(3)	Day (4)	Night (5)	(6)
Str.1	27.65	61.86	2.24	18.13	48.83	2.69
Str.2	39.61	197.33	4.98	25.54	116.10	4.55
Str.3	15.75	255.36	16.21	10.61	133.53	12.59
Str.4	150.42	487.41	3.24	91.98	284.20	3.09

1/ See: G.P. Bazigos - Applied Fishery Statistics, FAO-FIPS/T.135, P. 33-36

2/ The sample observations of Stratum 5 were insufficient for this kind of analysis

Figure 5.2.1 Estimated percentage distribution of the survey area into fish area (area with dots) and remaining area (area with lines), by stratum and for day and night periods of time

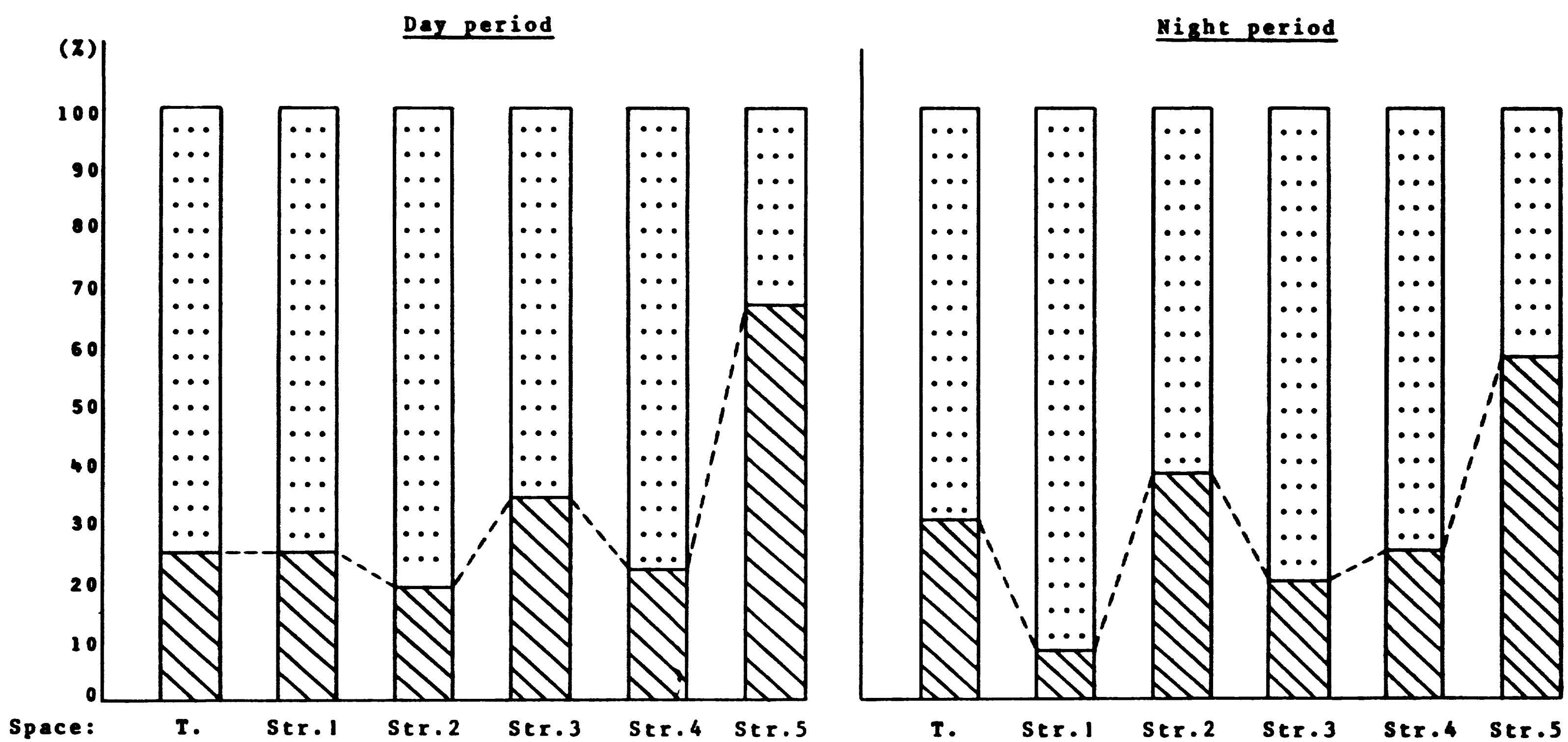
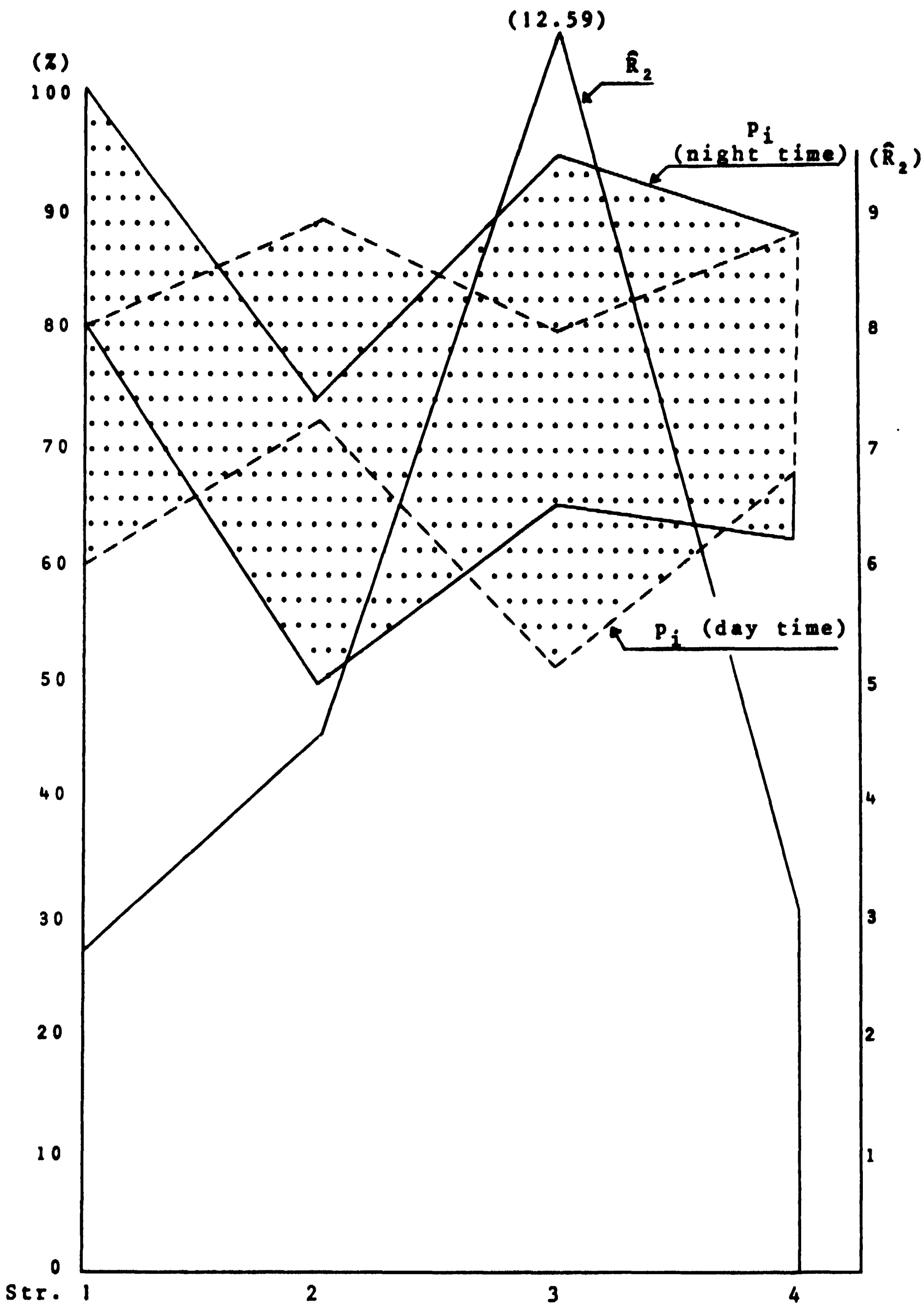


Figure 5.3.1 Trends in space of the magnitudes p_i (confidence interval P = 95%) and $\hat{R}_2$



The tabulated data (Table 5.3.1) indicate that the estimated level of dispersion of pelagic fish expressed by the values of $\hat{d}_1$ is higher during the night period than during the day period, and that there is a positive relationship between the abundance of fish ($\bar{y}$) and level of dispersion of fish ($\hat{d}_1$).

Another point of interest in this kind of analysis is the high variability in space of $\hat{R}_2$, column 6 in Table 5.3.1 ($\hat{R}_2$ = estimated ratio of the abundance of fish between night and day periods). Other things being equal, based on two samples selected from the same population, one would expect that if the values of p_i do not differ significantly in space (between strata), then the estimated values of $\hat{R}_2$ should follow a regular pattern. This, however, is not the case in strata 2,3, and 4, in which the hypothesis, Ho : $P_2 = P_3 = P_4$ is valid. A graphical presentation of the estimated trends in space of the magnitudes p_i and $\hat{R}_{2i}$ is given in Figure 5.3.1.

5.4 A graphical presentation of the dispersion pattern

It is a common practice in the field of applied fishery statistics to use charts for presenting and explaining numerical data. In our case, charts in oblique projection are used to portray the dispersion pattern of pelagic fish in Lake Tanganyika.

A first assessment of the sample data of the echo survey indicates that, very roughly, we can establish three sets of water-roads in the lake:

Water-road 1 - (L_1): It covers the east littoral zone of the lake

Water-road 2 - (L_2): Open waters

Water-road 3 - (L_3): It covers the west littoral zone of the lake

Figures 5.4.1 to 5.4.10, prepared for day and night periods on a stratum basis and for the lake as a whole, are based on the sample average integrator readings per ESDU ($\bar{y}$). The diagrams allow an optical comparison of the magnitude $\bar{y}$ vertically, between tracks within strata and between strata for the lake as a whole, and horizontally between water-roads. The values of the bars of L_1 can be simply read on the $\bar{y}$-axis, whereas the values of the bars of L_2 and L_3 can be read on the $\bar{y}$-axis only by taking into account the correspondingly higher drawn axes of L_2 and L_3. The numbers in the charts are the values of the estimated specific indices expressing relative changes of $\bar{y}$ between the established water-roads.

A visual assessment of the prepared chart-units leads to the following conclusions:

(a) Night measurements: There is consistency in the portrayed distribution of fish in space and L_2 can be considered as the modal area of the distribution.

(b) Day measurements: The consistency of the portrayed distribution of fish in space is a function of the recorded abundance of fish. Low abundance of fish fails to portray any clear dispersion pattern of fish (Str.3), whereas high abundance portrays a distribution similar to that of the night period1/ (Str.4).

1/ See also: Chapman, D. (1974) - Distribution of biomass of pelagic fish in Lake Tanganyika, FAO-URT/71/012/7, Working Paper No.11

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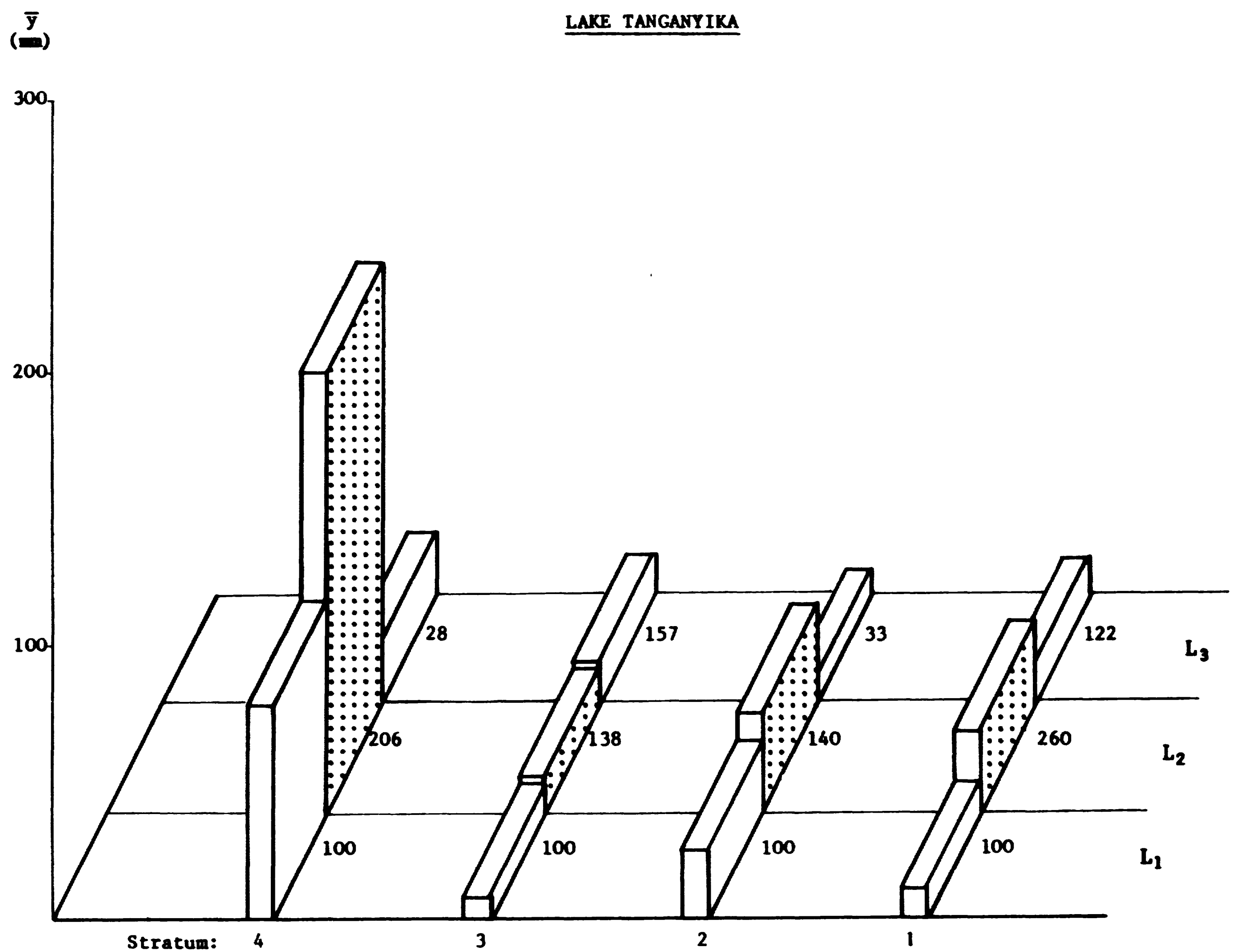

Figure 5.4.1 Dispersion pattern of fish for the lake as a whole: Day period

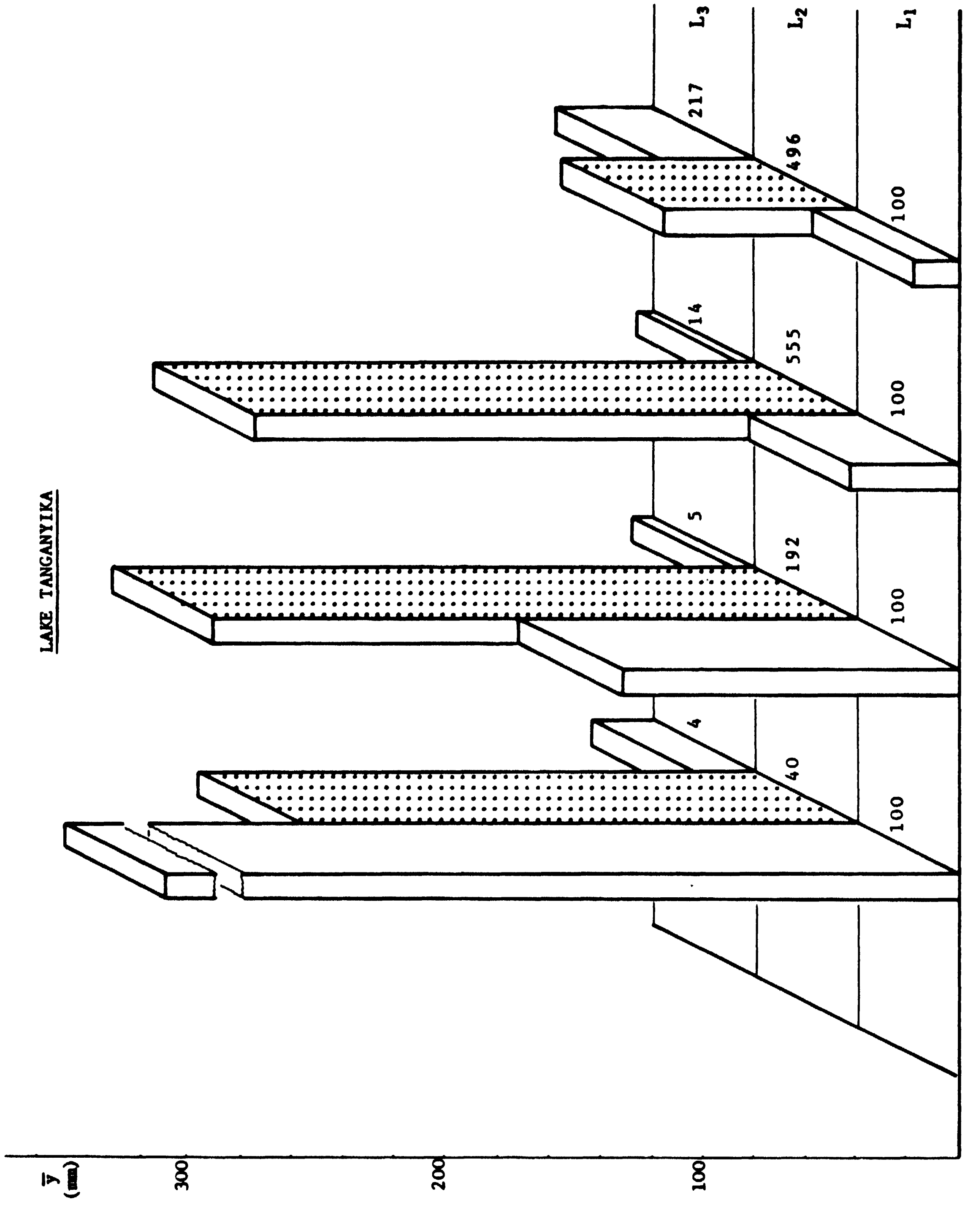

Figure 5.4.2 Dispersion pattern of fish for the lake as a whole: Night period

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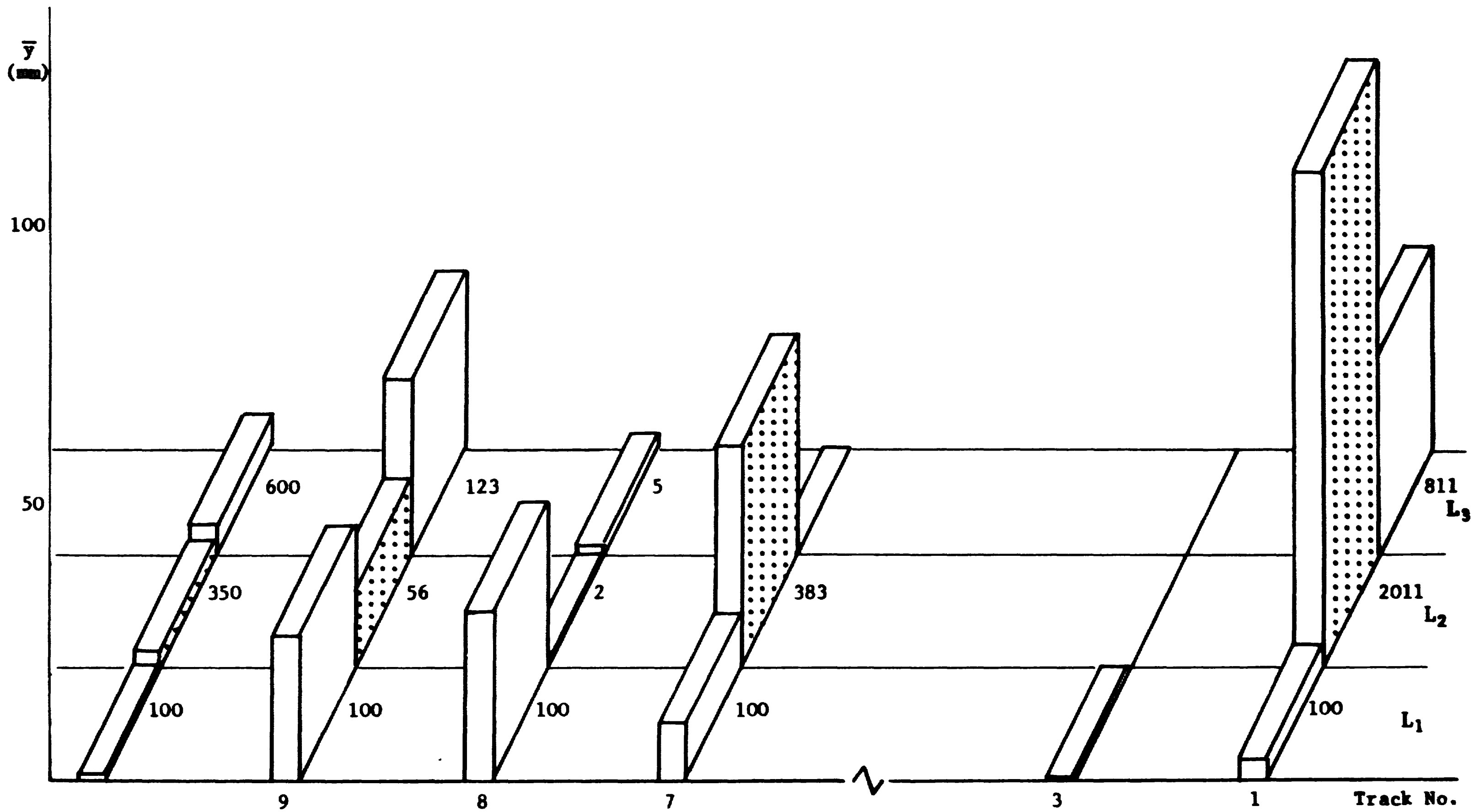

Figure 5.4.3 Dispersion pattern of fish within Stratum 1: Day period

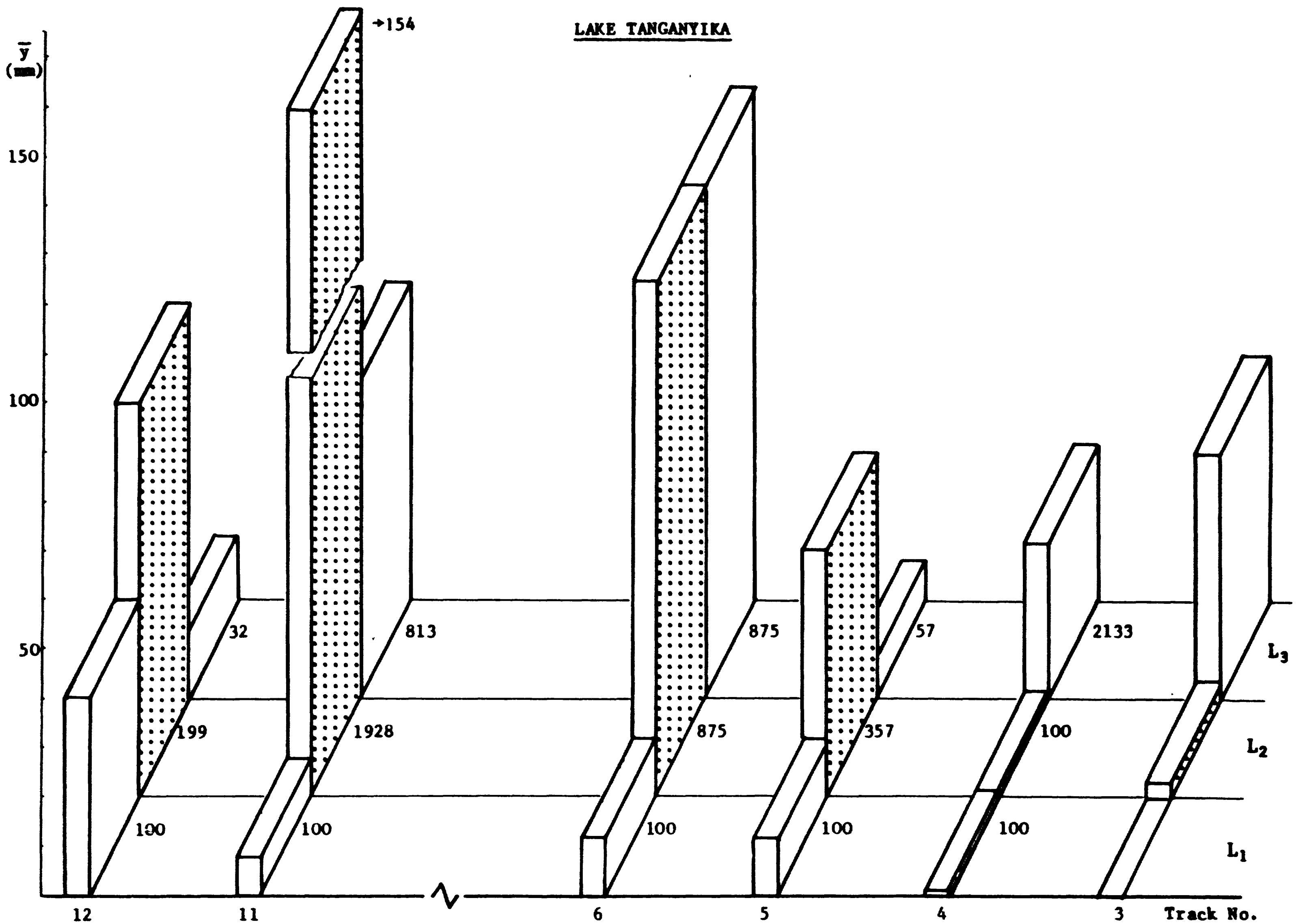

Figure 5.4.4 Dispersion pattern of fish within Stratum 1: Night period

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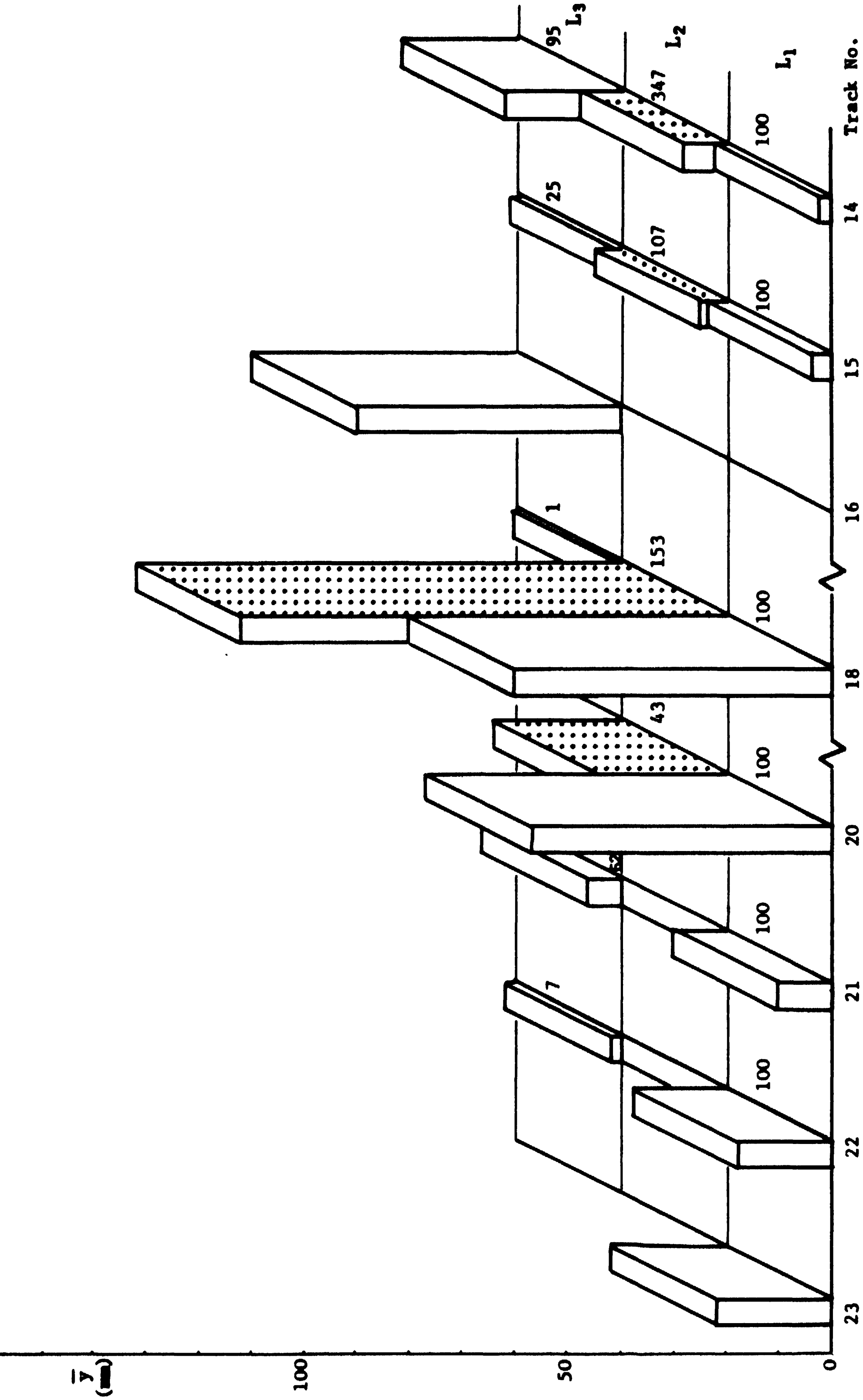

Figure 5.4.5 Dispersion pattern of fish within Stratum 2: Day period

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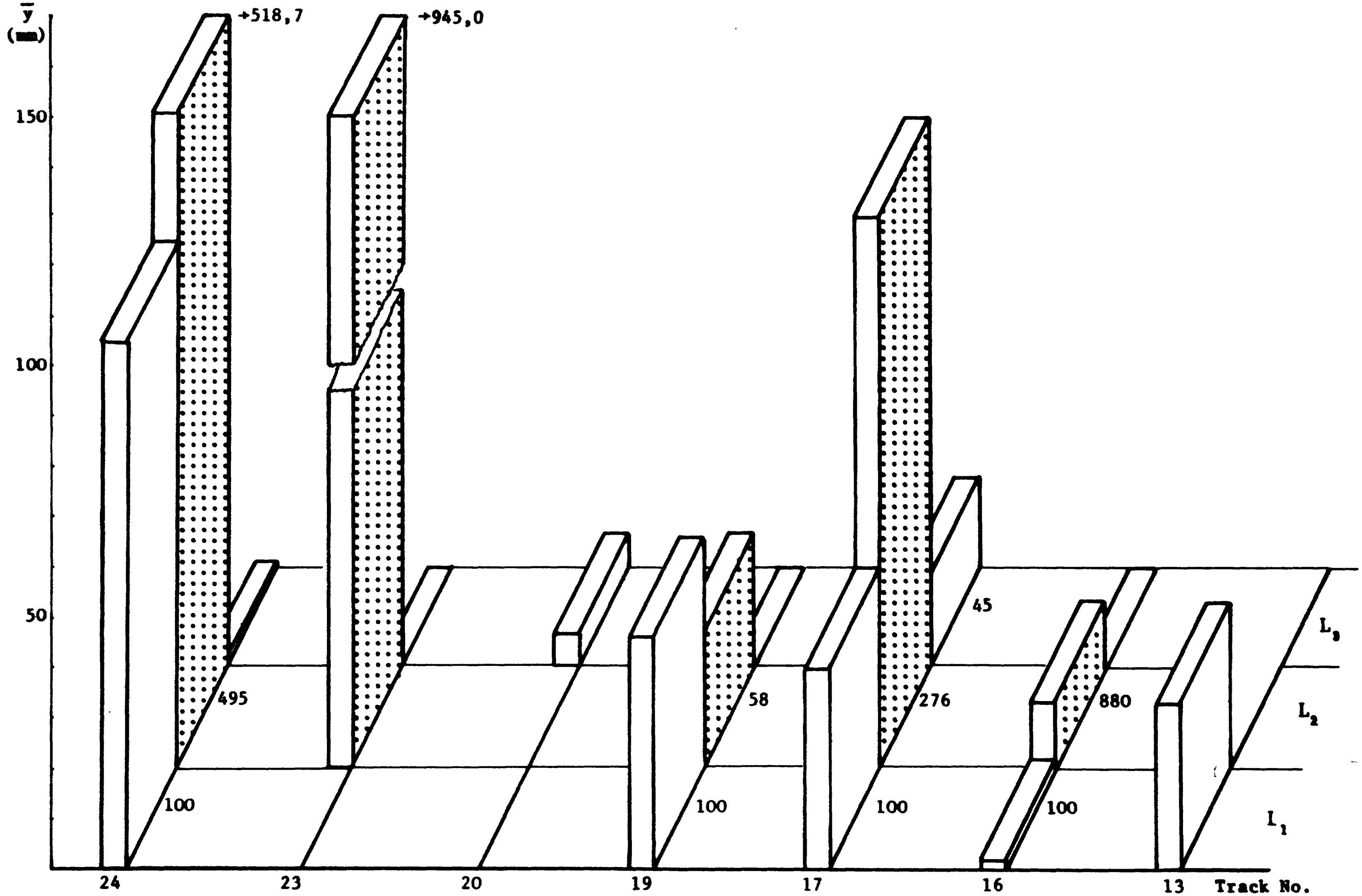

Figure 5.4.6 Dispersion pattern of fish within Stratum 2: Night period

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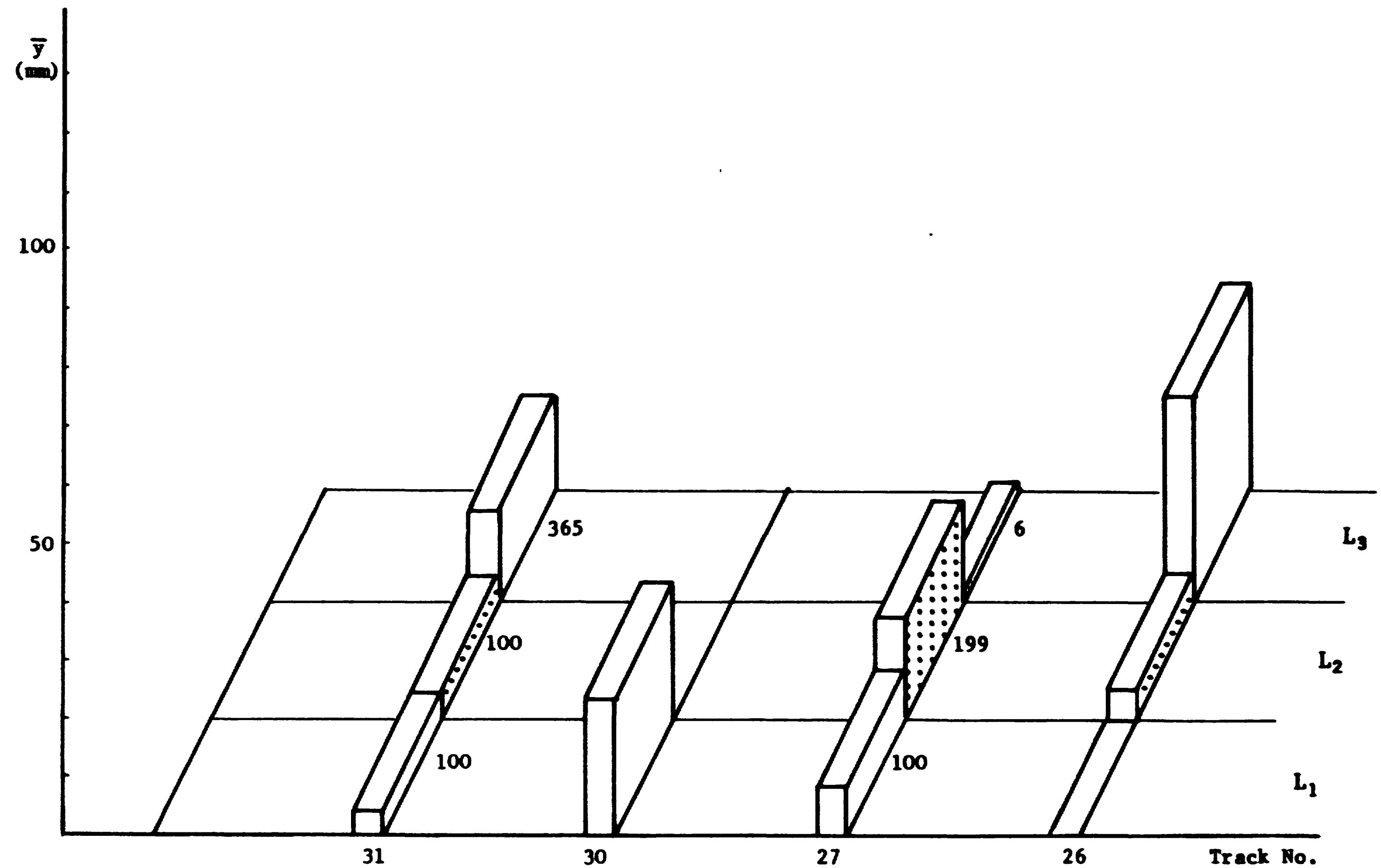

Figure 5.4.7 Dispersion pattern of fish within Stratum 3: Day period

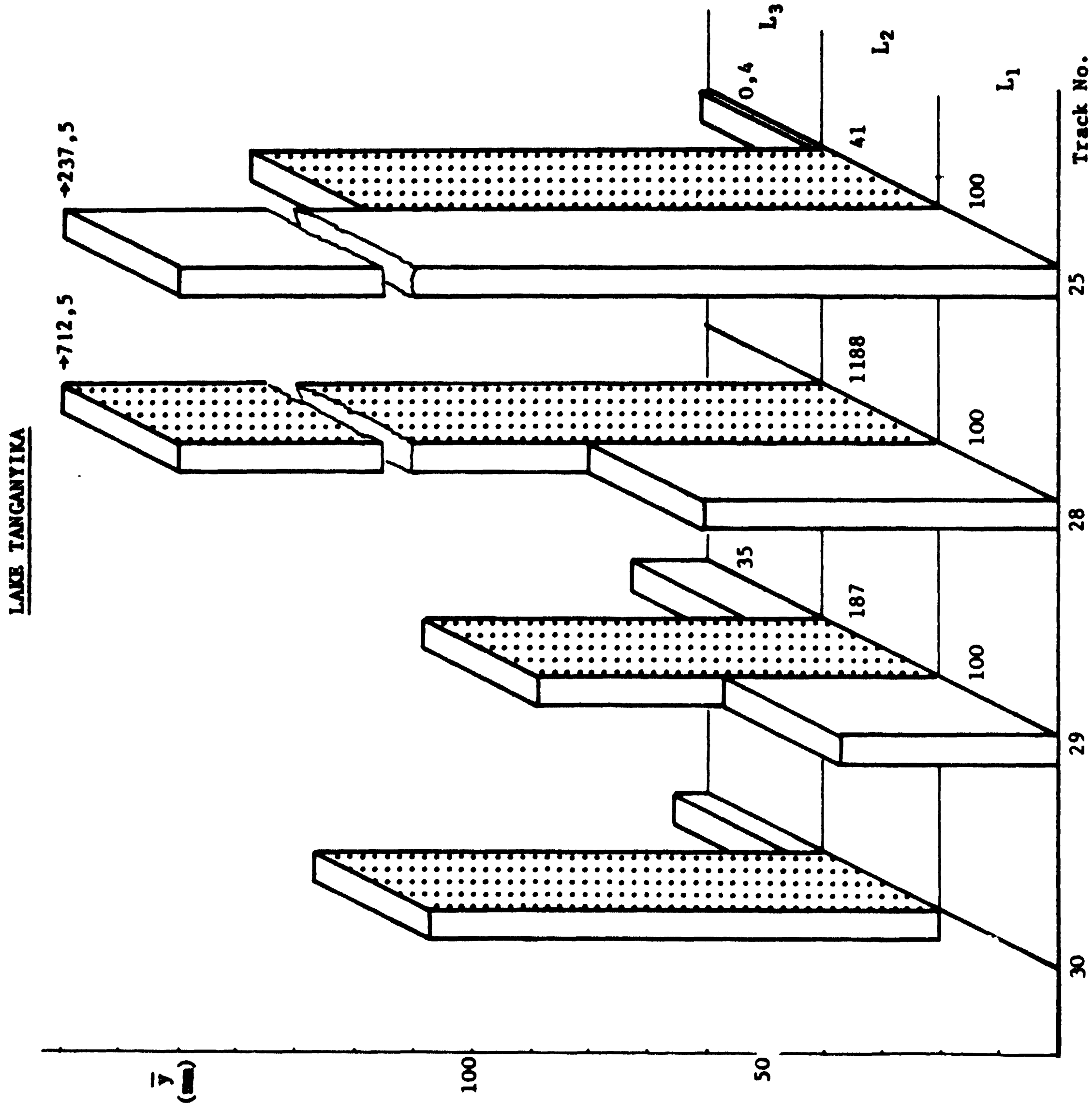

Figure 5.4.8 Dispersion pattern of fish within Stratum 3: Night period

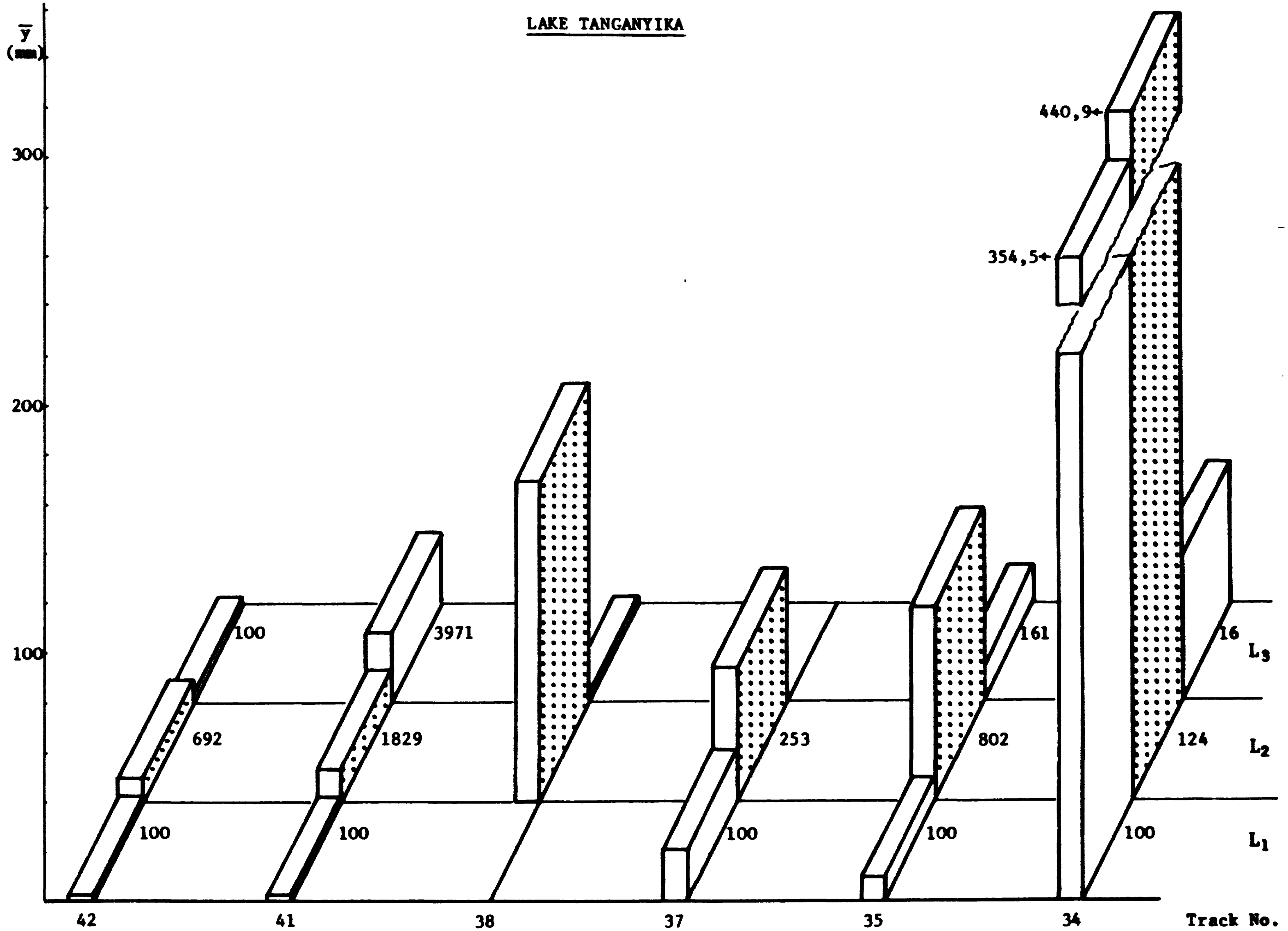

Figure 5.4.9 Dispersion pattern of fish within Stratum 4: Day period

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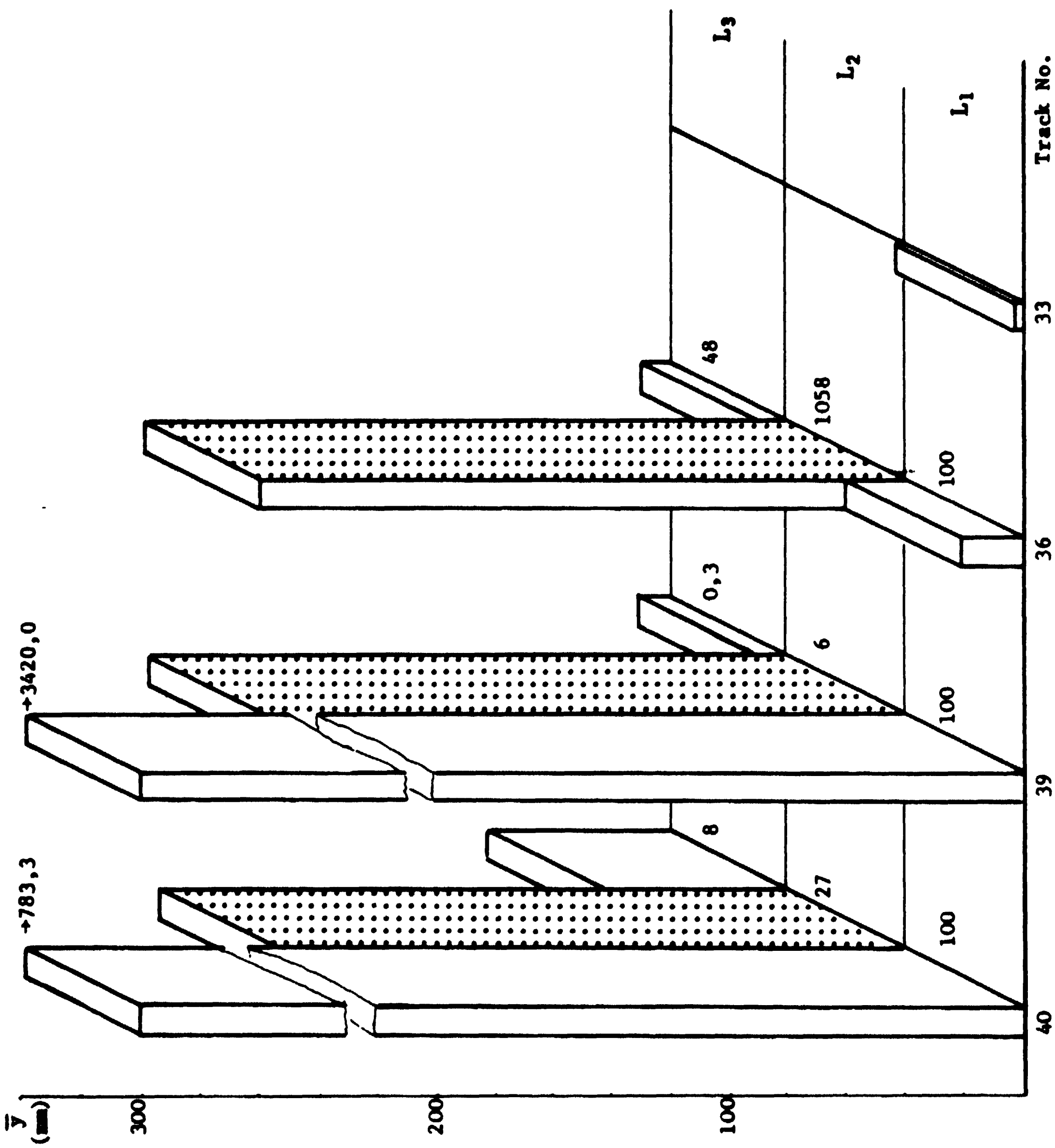


Figure 5.4.10 Dispersion pattern of fish within Stratum 4: Night period

CHAPTER 6: TYPE AND SOURCES OF ERRORS IN ECHO SURVEYS[1/]

6.1 Introduction

In this chapter we discuss the various types of errors (sampling errors and non-sampling errors) inherent in the survey system of an echo survey and their effect on the results of the survey. It is believed that the direction and size of the various types of errors of the survey (one occasion) is a function of the diel behavioural pattern of the surveyed population.

6.2 Sources and causes of errors

From a sampling point of view, detailed knowledge of the possible sources and causes of errors of a large-scale survey will help to minimize their operation through the adoption of appropriate survey procedures and techniques. It is safe to assume that in echo surveys certain factors have been left free to operate and form errors in the results of the surveys. Looking at the data of the main echo survey in Lake Tanganyika from the point of view of distortion caused by the error factors, I prepared the block diagram given below (Figure 6.2.1). I have divided the spectrum of potential errors into two domains, i.e. (1) External errors (of statistical sampling nature); and (2) Internal errors (non-sampling errors and technical errors). Specifically, external errors are due to the level of efficiency of the line sample of the survey which is a function of the type of spatial distribution of the surveyed population (see also section 6.3). Internal errors are attributed to the behaviour pattern of the surveyed population and measuring procedure of the survey.

6.2.1 Internal errors in echo surveys

The main sources of non-sampling errors and technical errors in echo surveys have been grouped into four categories by taking into account the nature of the source of errors (Figure 6.2.1), i.e:

2.1 Coverage errors of type-1

2.2 Coverage errors of type-2 / Avoidance errors

2.3 Measurement errors

2.4 Other kind of errors

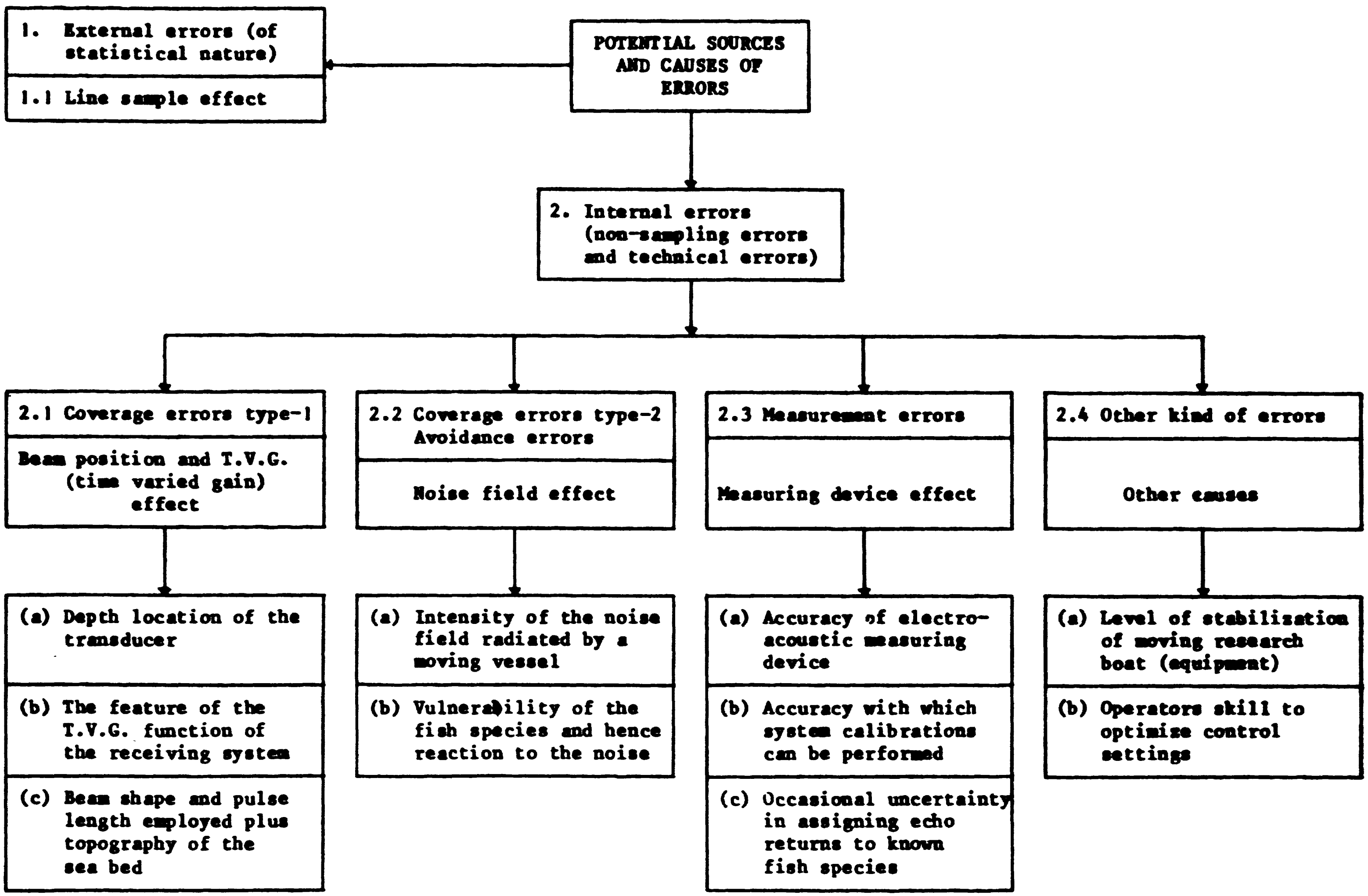
Further, within each of the above main sources of errors, I recorded the various factors which operate and form the respective errors (Figure 6.2.1). Specifically:

2.1 The coverage errors of type-1 originate primarily from limitations regarding the minimum depth at which the acoustic beam produces quantifyable fish echoes. This depth usually ranges from 5-7 m and is determined by two factors: (i) The depth location of the transducer which, for hull mounted transducers, may vary between 2 and 4 m, depending on vessel type; and (ii) The feature of the T.V.G. (time varied gain) function of the receiving system which precludes an efficient signal receiption for an initial time period corresponding to a range of 3 m immediately below the transducer.

Secondly, there are coverage errors (usually not very serious when dealing with pelagic species) which depend on the beam shape and pulse length employed, plus the topography of the sea bed.

1/ I would like to acknowledge that I received very useful comments from K.A. Johannesson (FAO-Fishery Resources Officer) during the preparation of this chapter

Figure 6.2.1 Potential errors of echo surveys



2.2 The magnitude of the coverage errors of type-2/ avoidance errors is apparently a function of a number of factors, such as:

(i) The intensity of the noise field radiated by a moving vessel

(ii) The vulnerability of the fish species and hence reaction (fright?) to this noise, which can be conceived as a disturbance-zone of a high level near the surface but diminishing with increased depth

(iii) The degree to which this zone penetrates the natural environment of the species concerned, e.g. those which typically ascend and form near-surface shoals during day time, or species that always remain relatively near the surface simply because of shallow water depths

2.3 The measurement errors are attributed to the inaccuracy of the electroacoustic measuring device and their overall magnitude largely depends on the accuracy with which system calibrations can be performed.

2.4 Other kind of errors are those related to:

(i) The fact that the vessel is pitching, rolling and tossing, etc., and the transducer (when stabilized) has to move with it

(ii) The operators skill to optimize control settings

6.3 Differences between day and night observations

Directly connected with the effect of the various types of errors described above, are the observed differences between day and night measurements1/. In spite of the standardization of the field operations of the survey, the magnitude of external errors is higher during the day time rather than during the night. Imagine someone at a cross-road, without traffic lights, and he crosses the road straight away (instantaneous reference period) without taking any precaution. From a mathematical point of view, the probability (P) of the person crossing the road being hit by a car is a function of the level of flow of cars. If there is no flow of cars, the value of P is equal to zero. If there is a continuous flow of cars the value of P is equal to 1, $0<P<1$. Furthermore, if the traffic flow per unit of time is given the probability of a person crossing the road and being hit by a car is a function of the "packing" pattern of cars. This in turn means that with an instantaneous reference period in the survey system of our survey, the level of efficiency of the line sample is a function of the packing pattern of moving fish. The efficiency of the sample is higher when the packing pattern of fish is expressed by layers of fish (night time observations) rather than when it is expressed by shoals of fish (day time observations). External errors lead to an under-estimation of the surveyed magnitude in day time.

It has been observed that during the survey period fish biomass tended to concentrate closer to the surface during the day rather than during the night. This in turn means that the magnitude of the internal errors of type 2.1 and 2.2, i.e. coverage errors of type-1 and coverage errors of type-2 and avoidance errors are bigger in day time rather than in night time. These errors lead to an under-estimation of the surveyed magnitude in day time.

1/ The estimated average overall ratio between night and day measurements was 4:1

Finally, it has been indicated that the measuring accuracy of the device is better when we are dealing with layers of fish (night time measurements) rather than with shoals of fish (day time measurements).

From the foregoing discussion one can see that the joint effect of the various sources of error (internal and external) are responsible for the observed differences between day time and night time measurements.

To get an idea of the joint effect of the various sources of errors on the accuracy of the sample estimates, I prepared Figure 6.3.1. In the diagram1/ there are graphical presentations of the confidence intervals (P = 95%) for $\bar{Y}_h$ (population average integrator readings per ESDU) for day and night and within the established strata. A visual assessment of the respective charts indicates that:

Str.3	There are highly significant differences between day and night observations
Str.2	There are significant differences between day and night observations
Str.1,4	The established confidence intervals are overlapping

The observed inconsistency in the level of statistical significance between day and night estimates of the surveyed magnitude portrays the unknown weighting system on the basis of which the various sources of errors affect the sample observations.

1/ For calculations the sample data of the horizontal tracks only were taken into account

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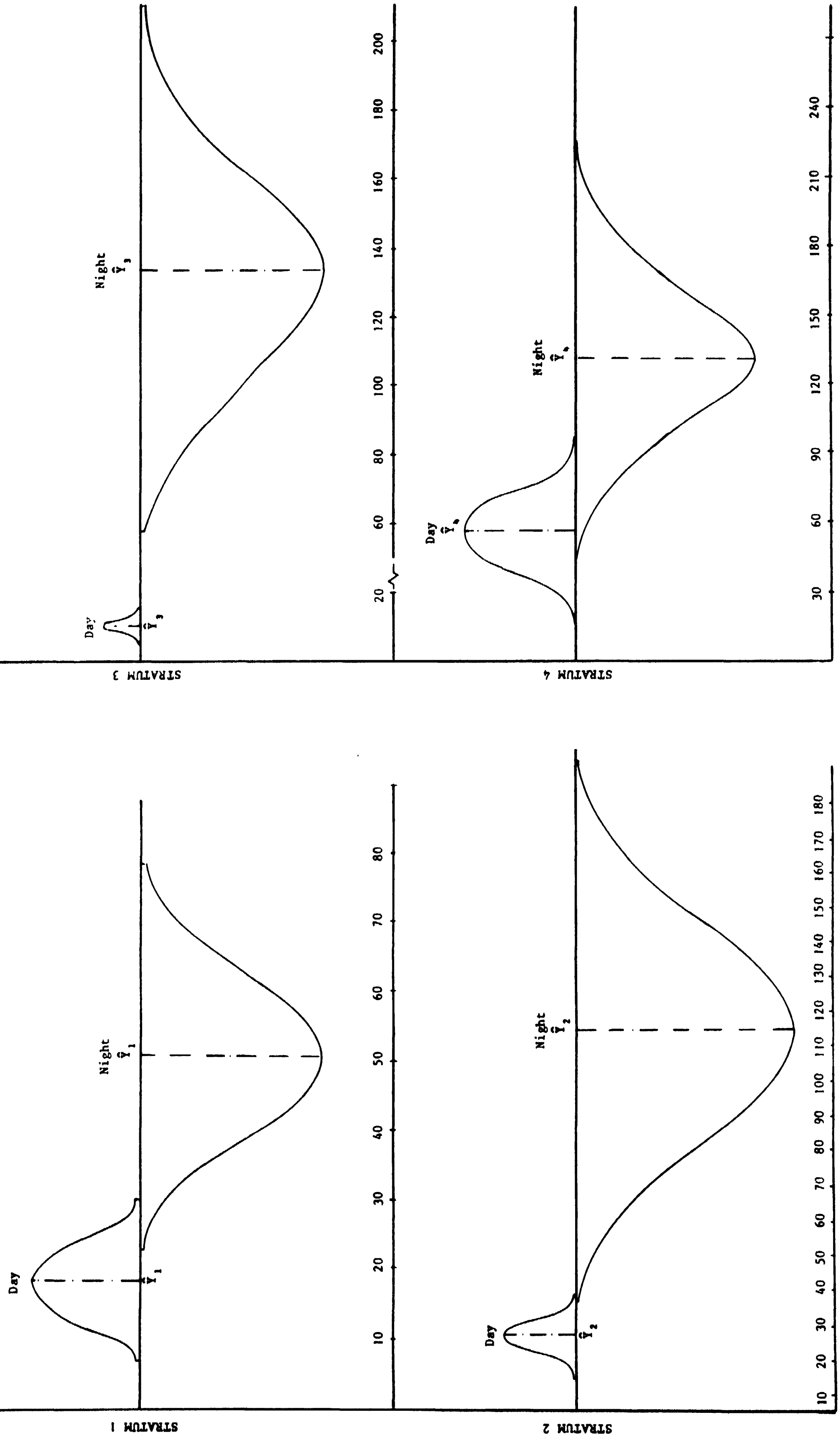
Figure 6.3.1 Estimated confidence intervals for $\bar{Y}_h$ (P = 95%) by stratum and by day and night periods



CHAPTER 7: ESTIMATION OF POPULATION VALUES

7.1 Introduction

In this chapter I determine the estimators which can be used to calculate the surveyed population values (see Figure 7.2.1) and their sampling errors[1]. Also, the relative efficiency of the different estimation procedures is discussed.

7.2 Method of collapsed strata

One method which can be used to estimate the population values is the method of collapsed strata. For the application of the method we should extend the stratification of the lake to the point where the sample contains only one track in each sub-stratum. For our purposes, and in order to simplify the analysis of the available sample observations, a sub-stratum is formed by considering the water up to α miles on either side of a given sample track[2], i.e. α equals three miles for the northern part of the lake, and five miles for the southern part. In this method, in which a track has been selected from each sub-stratum, the sampling variance is estimated by pairing the sub-strata to form collapsed strata. Since the precision of the method increases if the pairing can be so arranged (before the collection of data) that the strata forming the pair are about equal in size (total or average of the survey variate), I formed collapsed strata by pairing adjacent sub-strata within the existing strata.

Notation[3]:

i : stands for a given sub-stratum

k : stands for a given collapsed stratum

y_{hi} : total integrator reading (mm) of the sample track in the i^{th} sub-stratum, within the h^{th} stratum

n_{hi} : number of ESDUs in the sample track

$_\beta\hat{W}_{hi} = \hat{C}A_{hi}\left(\frac{y_{hi}}{n_{hi}}\right)$: Estimated total biomass (t) in the i^{th} sub-stratum

$_\beta\hat{W}_h = \sum_i {_\beta\hat{W}_{hi}}$: Estimated total biomass (t) in the h^{th} stratum

$_\beta\hat{W} = \sum_h {_\beta\hat{W}_h}$: Estimated total biomass (t) for the lake as a whole

---

1/ Estimators have been developed under the following two assumptions: (a) in the field operations of the survey, the same shoals/layers have not been covered more than once over the successive sample transects of the survey (see also Chapter 4); and (b) the calibration method of the survey was free of errors. The statistical efficiency of the calibration method of echo surveys will be discussed in another paper. Also, in the same paper, I will determine the estimator of the overall variance of the sample estimates and its analysis into components. For estimation purposes the night sample observations of the horizontal tracks of the survey were used.

2/ Under ideal conditions the lake should be stratified in advance into a number of limnological zones (horizontally) and one sample track should be selected randomly within the established sub-strata (limnological zones).

3/ See also section 3.5

$v({}_{\beta}\hat{W}_{hk}) = ({}_{\beta}\hat{W}_{h1} - {}_{\beta}\hat{W}_{h2})^2$: Estimated variance of the estimated total biomass in the k^{th} collapsed stratum[1/]

$v({}_{\beta}\hat{W}_h) = \sum_k v({}_{\beta}\hat{W}_{hk})$: Estimated variance of the estimated total biomass in the h^{th} stratum

$v({}_{\beta}\hat{W}) = \sum_h v({}_{\beta}\hat{W}_h)$: Estimated variance of the estimated total biomass for the lake as a whole

By applying the estimators established above on the sample data estimates were calculated of the survey characteristics. The Table below (Table 7.2.1) shows both the estimates of the total biomass of pelagic fish by stratum, major stratum and for the lake as a whole, and of the sampling variance, sampling error and relative sampling error of the estimated magnitudes. Also, in columns 7 and 8 of the Table, the confidence limits (P = 95%) of the parameters under estimation are given[2/]. Major strata were formed by grouping together the established strata. Specifically, Major Stratum I consists of Strata 1,2,3 (northern part of the lake with sample transects at 6 n mi intervals) and Major Stratum II consists of Strata 4,5 (southern part of the lake with sample transects at 10 n mi intervals).

Table 7.2.1 Point estimates and estimated confidence intervals for the total biomass of pelagic fish in the lake (P = 95%) by stratum, major stratum, and for the lake as a whole (in metric tons)

Major Str. (1)	Str. (2)	Point estimates (3)	Sampling variances (4)	Sampling errors (5)	Relative sampling errors (6)	Conf. intervals (95%) Lower limit (7)	Upper limit (8)
TOTAL:		3051953	438664796780	662317	21.70	1612698	4491168
I		997717	22145267449	148812	14.92	524198	1471236
	1	88686	104572816	10226	11.53	56147	121225
	2	474930	17039000997	130533	27.48	*	*
	3	434101	5001693626	70722	16.29	*	*
II		2054236	416519529331	645383	31.42	*	*
	4	2015940	416256862482	645179	32.00	*	*
	5	38296	262666849	16207	42.32	*	*

* The sampling distribution of the estimate is highly dispersed

1/ The quantity $v({}_{\beta}\hat{W}_{hk})$ overstates the true variance of ${}_{\beta}\hat{W}_{hk}$. The overstatement depending upon the extent to which strata forming the same pair differ with respect to their totals

2/ One should remember that the size of the total biomass is not fixed over time and that the tabulated data (Table 7.2.1) gives the size of the estimated magnitudes at a given period of time, i.e. November 1973

An assessment of the results of the method of collapsed strata (Table 7.2.1) leads to the following conclusions:

(a) The strata variance is a function of the level of abundance of pelagic fish and proportional allocation of the sample tracks between strata is not a beneficial method. In our case the optimum allocation must be used.

(b) The sample size in Major Stratum II (sample transects at 10 n mi intervals) should be considered as a small sample for the survey and this is reflected in the estimated relative sampling error.

(c) The level of precision achieved in Major Stratum II has affected the level of reliability of the sample estimates of total biomass of pelagic fish for the lake as a whole.

(d) The level of precision of the estimated magnitude for Major Stratum I should be considered as good.

(e) The precision of the method of collapsed strata would be highly improved if the sample of the survey in the strata with a high abundance of fish (specifically Str. 2 and 4) was based on transects at 3 n mi intervals and in the remaining ones at 6 n mi intervals.

The Table below (Table 7.2.2) gives the point estimates based on the methods of collapsed strata and of the "graphical method" used in the survey[1/].

Table 7.2.2 Point estimates of the total biomass of pelagic fish in Lake Tanganyika based on various estimation procedures (in metric tons)

Major Str.	Str.	Point estimates		Point estimates (%)			
		Graphical method	Method of collapsed strata	Graphical method		Method of collapsed strata	
TOTAL:		2600000	3051953	100.00		100.00	
I		820000	997717	31.54	100.00	32.69	100.00
	1	120000	88686		14.63		8.89
	2	360000	474930		43.90		47.60
	3	340000	434101		41.47		43.51
II		1780000	2054236	68.46	100.00	67.31	100.00
	4	1700000	2015940		95.51		98.13
	5	80000	38296		4.49		1.87

1/ For estimation purposes in the main survey, the day time sample data were multiplied by 4 (overall correcting factor) before they matched with the night time sample data

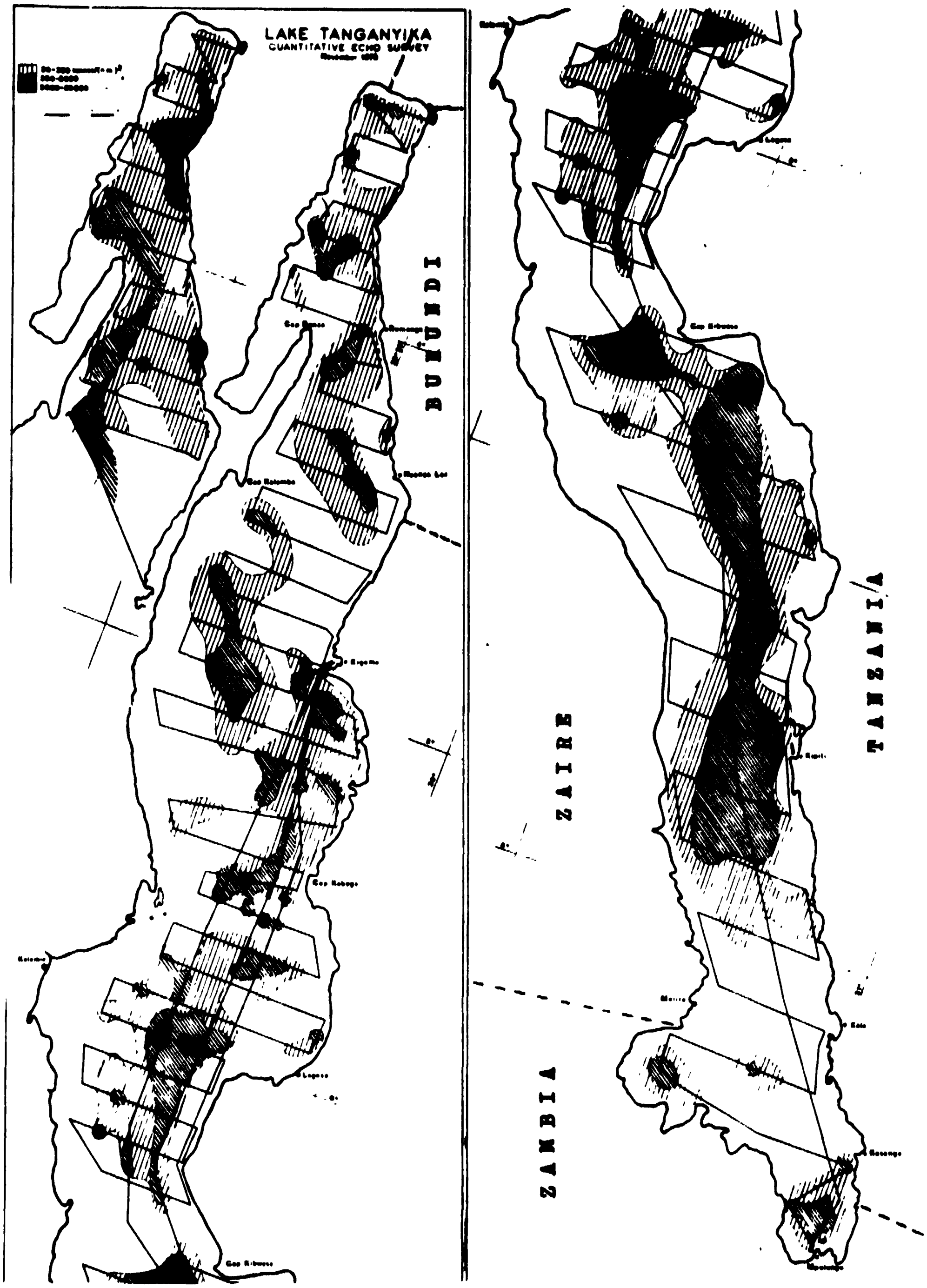


Figure 7.2.1 Relative abundance of pelagic fish stocks in Lake Tanganyika

In the above Table the point estimates based on the two estimation procedures do not show any statistically significant differences at P = 95%.

In Figure 7.2.1 the qualitative results of the echo survey based on the graphical method are portrayed.

7.3 A second estimator for total biomass of pelagic fish

Another method to estimate the total biomass of pelagic fish in the lake can be developed by considering the sample tracks (horizontal tracks) within the established strata as units of random samples selected from the survey population. Also, since we are dealing with sample tracks of various sizes, as far as the number of ESDUs is concerned, the following estimator can be used for the calculation of the surveyed population total:

Notation:

j : The suffix j stands for a given track

h : The suffix h stands for a given stratum

y_{hj} : Total integrator reading (mm) of the j^{th} track in stratum h

n_h : Number of sample tracks in the h^{th} stratum

$_\beta\hat{W}_{h(j)} = \hat{C}A_h \dfrac{y_{hj}}{n_{hj}}$: Estimated total biomass in the h^{th} stratum based on the j^{th} track and n_{hj} number of ESDUs in the j^{th} track

$_\beta\hat{W}_h = \dfrac{1}{n_h} \sum_j {_\beta\hat{W}_{h(j)}}$: Estimated total average biomass in the h^{th} stratum

$_\beta\hat{W} = \sum_h {_\beta\hat{W}_h}$: Estimated total biomass for the lake as a whole

$$v(_\beta\hat{W}_h) = \frac{1}{n_h(n_h-1)} \left\{\sum_j {_\beta\hat{W}^2_{h(j)}} - \frac{(\sum_j {_\beta\hat{W}_{h(j)}})^2}{n_h}\right\}$$

: Estimated variance of the estimated total biomass in the h^{th} stratum

$v(_\beta\hat{W}) = \sum_h v(_\beta\hat{W}_h)$: Estimated variance of the estimated total biomass for the lake as a whole

An assessment of the calculated precision of the estimated magnitude indicates that the method is not the proper one for our purposes. The estimated relative sampling error for the lake as a whole was of the level of about 50 percent.

7.4 A third estimator for total biomass of pelagic fish

An assessment of the dispersion pattern of pelagic fish in the lake (Chapter 5) indicates that the precision of the estimates based on unstratified sample (section 7.3) can be highly improved if the method of stratified sampling based on interpenetrating sub-samples is used. In such a case the lake should be divided into not less than 30 strata by horizontal partition lines and into 3 main sets of waterroads, dividing the lake lengthwise. Two sample tracks should be selected within the established cells (systematic selection, in order to simplify the field operations). It is expected that if the proper control characteristics for stratification should be employed, this method would provide much more precise estimates than those of the method described in section 7.3.

7.5 The relative efficiency of precision

In sampling techniques, the relative efficiency of precision of two different methods of sampling based on the same type of sampling unit may be defined as the reciprocal of the ratio of the sampling variances of the estimates given by the two methods when the same number of units are taken. In our case estimates of the precision of the survey characteristic were calculated by using the method of collapsed strata (section 7.2) and the method of unstratified sampling (section 7.3). An estimate of the relative efficiency of the two methods for the lake as a whole is given by:

$$RE_{(pr)} = \frac{cv_2^2(_\beta\hat{W})}{cv_1^2(_\beta\hat{W})} = \frac{50^2}{21.70^2} = 5.31$$

where, $cv_2(_\beta\hat{W})$: Is the coefficient of variation of the estimated magnitude based on method 2 (section 7.3)

$cv_1(_\beta\hat{W})$: Is the coefficient of variation of the estimated magnitude based on method 1 (section 7.2)

An assessment of the estimated value of $RE_{(pr)}$ indicates that the method of unstratified sample provides five times the maximum variance of the estimated characteristic (total biomass of pelagic fish in the lake) as the method of collapsed strata.

APPENDIX I

A MATHEMATICAL MODEL FOR ACOUSTIC SURVEY IN LAKE TANGANYIKA

A MATHEMATICAL MODEL FOR ACOUSTIC SURVEY IN LAKE TANGANYIKA[1/]

Introduction

The main objective of this analysis was to attempt to fit a mathematical model to the empirical data of the survey in order to explain the spatial distribution of the fish stocks, and to obtain a suitable transformation to normalize the frequency distribution of the data in order to perform further statistical analysis such as regression and analysis of variance.

Methodology[2/]

For this analysis Stratum 2, the largest stratum was selected. The basic variable considered was y = integrator reading. Also, it was recognized that the fish move in schools at day time and in layers at night. Hence day time and night time data were treated separately.

Night data

Firstly, the night data were arranged in a frequency distribution table. The sample mean ($\bar{y}$) and variance (s^2) were then computed. The purpose was to compare the magnitudes of the two statistics $\bar{y}$ and s^2, to indicate which theoretical distribution might be a suitable model. Now for this type of population, three mathematical distributions are possible models, namely Positive Binomial ($\sigma^2 < \mu$), Poisson ($\sigma^2 = \mu$), and Negative Binomial ($\sigma^2 > \mu$), where μ, σ^2 are population mean and variance respectively; $\bar{y}$ was found to be 170.90, whilst s^2 was 141896.61. As $\bar{y}$ and s^2 are unbiased estimates of the population mean (μ) and variance (σ^2) and $s^2 > \bar{y}$, there is a strong indication that the negative binomial distribution is the most probable model. The chi-squared "goodness-of-fit" test was applied to ascertain the adequacy of the model, but unfortunately the relevant negative binomial distribution tables were not available. Assuming the model to be adequate, the spatial dispersion of the population could be described as a contagious distribution or clumped distribution. The negative binomial distribution is given by $p^k (1-q)^{-k}$, where $\frac{1}{q} = \frac{\mu}{k}$, $q = 1-p$ and k is estimated by $\frac{\bar{y}^2}{s^2-\bar{y}}$. The individual terms are given by:

$$P_{(x)} = \left(1+\frac{\mu}{k}\right)^{-k} \frac{(k+x-1)!}{x!(k-1)!} \left(\frac{\mu}{\mu+k}\right)^x$$

where $P_{(x)}$ is the probability of x individuals in a sampling unit. In our case, $k = 0.21$.

1/ Prepared by S. Adjei, FAO Fellow, Department of Fisheries, who was working under my supervision during the period 20 June to 12 July 1974 to round out two-year fellowship studies in the United States, Canada and Italy

2/ See also: G.P. Bazigos - Applied Fishery Statistics, FIPS/T.135, p. 42-50

Logarithmic transformation

The next step was to find a suitable transformation to normalize the frequency distribution. Now for a negative binomial distribution with k less than two, the usual transformation is the logarithmic transformation. In particular the zero counts in the data suggested the transformation $Z = \log_e (y+1)$ as the appropriate transformation for normality. To test the validity of this transformation the chi-squared "goodness-of-fit" test was applied. The transformed data were assumed to follow the normal distribution whose mean and variance were estimated by the corresponding sample values. The expected frequencies (f_i') were obtained by considering the cumulative distribution tables of the normal distribution. The chi-squared statistic,

$$\chi^2 = \sum_{i=1}^{n} \frac{(f_i - f_i')^2}{f_i'}$$

was then computed to be 28.11, where f_i = observed frequencies, and n = number of frequency classes. Now, under the hypothesis of normality the above statistic follows a chi-squared distribution with n-3 degrees of freedom (df). Now n = 32, therefore df = 29. But the tabulated value for 95 percent level of $\chi^2_{(29)}$ = 42.56. Hence, it is concluded that at 95 percent confidence level the normal distribution fits our empirical data adequately. Hence the recommended transformation is $Z = \log_e (y+1)$. The analysis was repeated for the day time data and the negative binomial distribution was again found to be the appropriate mathematical model. Again the transformation log (y+1) was found to normalize the data. Except for two points out of 41 which could be treated as outliers all the remaining 39 points in the frequency distribution table satisfied the chi-squared criterion.

STRATUM 2 - NIGHT DATA

$$\text{Mean } \bar{y} = \Sigma\frac{fy_i}{n} = \frac{7007}{41} = 170.90$$

$$\text{Sample variance } s^2 = \frac{\Sigma(fy^2)-\bar{y}\Sigma fy}{n-1}$$

$$= \frac{6873361.00-1197496.30}{40}$$

$$= 141896.61$$

$$s^2 > \bar{y} = \text{negative binomial model}$$

$$\hat{k} = \frac{\bar{y}^2}{s^2-\bar{y}} = 0.21$$

$$\text{Log transformation } z = \log_e (y+1)$$

$$\text{s.d.} = \sqrt{s^2} = s = 376.69$$

Transformed data:

$$\bar{Z} = \Sigma\frac{\log_e (y+1)}{n} = 3.57$$

$$s_1^2 = \frac{\Sigma(fz^2)-\bar{z}\Sigma fz}{n-1} = 3.53$$

$$s_1 = \sqrt{3.53} = \underline{1.88}$$

ACOUSTIC SURVEY - LAKE TANGANYIKA

STRATUM 2 - NIGHT DATA

f	y	y^2	fy	fy^2	$Z = \log_e(y+1)$	Z^2f	Zf
1	0	0	0	0	0.00	0.00	0.00
2	2	4	4	8	1.10	2.42	2.20
2	4	16	8	32	1.61	5.18	3.22
1	5	25	5	25	1.79	3.20	1.79
1	6	36	6	36	1.95	3.80	1.95
2	7	49	14	98	2.08	8.33	4.16
2	10	100	20	200	2.40	11.52	4.80
1	11	121	11	121	2.48	6.15	2.48
1	14	144	12	144	2.56	6.55	2.56
1	16	256	16	256	2.83	8.01	2.83
1	20	400	20	400	3.04	9.24	3.04
2	28	784	56	1568	3.37	22.71	6.74
1	29	841	29	841	3.40	11.56	3.40
1	30	900	30	900	3.43	11.76	3.43
1	36	1296	36	1296	3.61	13.03	3.61
2	38	1444	76	2888	3.66	26.79	7.32
2	42	1764	84	3528	3.76	28.14	7.52
2	48	2304	96	4608	3.89	30.13	7.78
1	52	2704	52	2704	3.97	15.76	3.97
1	55	3025	55	3025	4.03	16.24	4.03
1	71	5041	71	5041	4.28	18.32	4.28
1	72	5184	72	5184	4.29	18.33	4.29
1	94	8836	94	8836	4.55	20.70	4.55
2	105	11025	210	22050	4.66	43.42	9.32
1	304	92416	304	92416	5.72	32.72	5.72
1	310	96100	310	96100	5.74	32.95	5.74
1	366	133956	366	133956	5.91	34.93	5.91
1	450	202500	450	202500	6.11	37.33	6.11
1	470	220900	470	220900	6.15	37.82	6.15
1	880	774400	880	774400	6.78	45.97	6.78
1	1170	1368900	1170	1368900	7.06	49.84	7.06
1	1980	3920400	1980	3920400	8.44	71.23	8.44
Total n = 41						688.08	153.18

Log transformation: $Z = \log_e(y+1)$

ACOUSTIC SURVEY - LAKE TANGANYIKA

STRATUM 2 - NIGHT DATA

y_i	f_i	x	$F_i(x)$	$nF_i(x)$	$f_i' = n(F_{i+1}-F_i)$	$(f_i-f_i')^2$	χ^2
0	1	-1.90	0.03	1.23	1.23	0.05	0.04
2	2	-1.31	0.10	4.10	2.87	0.76	0.26
4	2	-1.04	0.15	6.15	2.05	0.0025	0.00
5	1	-0.95	0.17	6.97	0.82	0.03	0.04
6	1	-0.86	0.19	7.79	0.82	0.03	0.04
7	2	-0.79	0.21	8.61	0.82	1.39	1.70
10	2	-0.62	0.27	11.07	2.46	0.21	0.09
11	1	-0.58	0.28	11.48	0.41	0.35	0.85
12	1	-0.54	0.29	11.89	0.41	0.35	0.85
16	1	-0.39	0.35	14.35	2.46	2.13	0.87
20	1	-0.28	0.39	15.99	1.64	0.41	0.25
28	2	-0.1063	0.45	18.45	2.87	0.76	0.26
29	1	-0.0904	0.46	18.86	0.41	0.35	0.85
30	1	-0.07	0.47	19.27	0.41	0.35	0.85
36	1	0.02	0.51	20.91	1.64	0.41	0.25
38	2	0.05	0.52	21.32	0.41	2.53	6.17
42	2	0.10	0.54	22.14	0.82	1.39	1.70
48	2	0.17	0.57	23.37	1.23	0.59	0.48
52	1	0.21	0.58	23.78	0.41	0.35	0.85
55	1	0.25	0.60	24.60	0.82	0.03	0.04
71	1	0.3776	0.64	26.24	2.05	1.10	0.54
72	1	0.3829	0.65	26.65	0.41	0.35	0.85
94	1	0.5212	0.70	28.70	2.05	1.10	0.54
105	2	0.58	0.72	29.52	0.82	1.39	1.70
304	1	1.14	0.87	35.67	6.15	26.52	4.31
310	1	1.1542	0.88	36.08	0.41	0.35	0.85
366	1	1.24	0.89	36.49	0.41	0.35	0.85
450	1	1.35	0.91	37.31	0.82	0.03	0.04
470	1	1.3723	0.92	37.72	0.41	0.35	0.85
880	1	1.71	0.96	39.36	1.64	0.41	0.25
1170	1	1.86	0.97	39.77	0.41	0.35	0.85
1980	1	2.59	1.00	41.00	1.23	0.05	0.04

$z = \log_e(y+1)$ $\quad x = \frac{z-\hat{\mu}}{\hat{\sigma}}$ $\quad \chi^2 = 28.11$

No. of frequency classes = 32

No. of parameters estimated from $N(\mu,\sigma^2)$ = 2

Therefore, d.f. = 32-2-1 = 29

Tabulated value of $\chi^2_{(29)}$ at 95 percent level = 42.56

Hence normal distributions fit adequately at 95 percent confidence level

MATHEMATICAL MODEL FOR ACOUSTIC SURVEY OF LAKE TANGANYIKA
STRATUM 2 - DAY DATA

Frequency distribution of integrator value (y)

f	19	18	5	6	2	2	1	2	5	1	1	4	2
y	0	1	2	3	4	5	6	7	8	9	11	12	13

f	1	2	2	3	3	1	1	1	2	1	3	1	1
y	14	17	18	22	24	26	30	31	32	36	42	45	46

f	2	1	2		1	1	1	1	1	1		1	
y	50	54	56		105	125	133	152	170	210		360	

$n = 102$

$\Sigma f y_i = 2369$

$\bar{y} = 23.23$

$\Sigma f y_i^2 = 306091$

$\bar{y} \Sigma f y_i = 55031.87$

$$\text{var}(y) = s^2 = \frac{\Sigma(fy^2) - \bar{y}\Sigma fy}{n-1}$$

$$= \frac{306091 - 55031.87}{101}$$

$$= 2485.74$$

i.e. $s^2 > \bar{y}$ = Negative binomial model

$\hat{k} = 0.22$ Required transformation $y \rightarrow \log(y+1)$

ACOUSTIC SURVEY - LAKE TANGANYIKA

STRATUM 2 - DAY DATA

y_i	f_i	x	$F_i(x)$	$nF_i(x_i)$	$f_i' = n(F_{i+1}-F_i)$	$(f_i-f_i')^2$	χ^2
0	19	-1.26	0.10	10.20	10.20	77.44	7.59*
1	18	-0.82	0.20	20.40	10.20	60.84	5.96
2	5	-0.55	0.29	29.58	9.18	17.47	1.90
3	6	-0.37	0.36	36.72	7.14	1.30	0.18
4	2	-0.23	0.41	41.82	5.10	9.61	1.88
5	2	-0.11	0.46	46.92	5.10	9.61	1.88
6	1	-0.01	0.50	51.00	4.08	9.49	2.33
7	2	0.08	0.53	54.06	3.06	1.12	0.37
8	5	0.15	0.55	56.10	2.04	4.24	2.08
9	1	0.22	0.59	60.18	4.08	9.49	2.33
11	1	0.34	0.63	64.26	4.08	9.49	2.33
12	4	0.39	0.65	66.30	2.04	3.84	1.88
13	2	0.44	0.67	68.34	2.04	0.00	0.00
14	1	0.48	0.68	69.36	1.02	0.00	0.00
17	2	0.60	0.73	74.46	5.10	9.61	1.88
18	2	0.6322	0.74	75.48	1.02	0.96	0.94
22	3	0.76	0.78	79.56	4.08	1.17	0.29
24	3	0.86	0.79	80.58	1.02	3.92	3.84
26	1	0.81	0.81	82.62	2.04	1.08	0.53
30	1	0.95	0.82	83.64	1.02	0.00	0.00
31	1	0.9741	0.83	84.66	1.02	0.00	0.00
32	2	0.99	0.84	85.68	1.02	0.96	0.94
36	1	1.06	0.86	87.72	2.04	1.08	0.53
42	3	1.1612	0.87	88.74	1.04	3.84	3.69
45	1	1.2064	0.88	89.76	1.02	0.00	0.00
46	1	1.2193	0.89	90.78	1.02	0.00	0.00
50	2	1.27	0.90	91.80	1.02	0.96	0.94
54	1	1.3225	0.9066	92.47	1.02	0.00	0.00
56	2	1.3419	0.9099	92.81	0.34	2.76	8.11*
105	1	1.74	0.9591	97.83	5.02	16.16	3.22
125	1	1.8580	0.9686	98.80	1.02	0.00	0.00
133	1	1.8967	0.9713	99.07	0.27	0.53	1.96
152	1	1.98	0.9761	99.56	0.49	0.26	0.53
170	1	2.0516	0.9798	99.94	0.38	0.38	1.00
210	1	2.19	0.9857	100.54	0.60	0.16	0.27
360	1	2.54	0.9945	101.44	0.90	0.01	0.01

$z = \log_e (y+1)$

$x = \frac{z-\hat{\mu}}{\hat{\sigma}}$

$n = 102$

$s_1 = \hat{\sigma} = 1.55$

$\chi^2 = 59.48$

$\chi^2 = 43.78$**

$\chi^2_{est} = 43.78$** $< \chi^2_{0.05,33} = 43.80$

** Estimated χ^2 without the two values indicated by asterisk

APPENDIX II

ESTIMATED DISPERSION MATRICES

($\hat{d}_1$: estimated Mean Difference of GINI)

xj	fj	xi: fi:	0 8	1 4	2 2	3 3	4 1	5 1	6 1	9 1	17 1	20 1	21 1	26 2	32 1	44 1	52 1	85 1	96 1	120 1	Tj	fj	Tj fj
0	8		0 0	1 4	2 4	3 9	4 4	5 5	6 6	9 9	17 17	20 20	21 21	26 52	32 32	44 44	52 52	85 85	96 96	120 120	580	8	4640
1	4			0 0	1 2	2 6	3 3	4 4	5 5	8 8	16 16	19 19	20 20	25 50	31 31	43 43	51 51	84 84	95 95	119 119	556	4	2224
2	2				0 0	1 3	2 2	3 3	4 4	7 7	15 15	18 18	19 19	24 48	30 30	42 42	50 50	83 83	94 94	118 118	536	2	1072
3	3					0 0	1 1	2 2	3 3	6 6	14 14	17 17	18 18	23 46	29 29	41 41	49 49	82 82	93 93	117 117	518	3	1554
4	1						0 0	1 1	2 2	5 5	13 13	16 16	17 17	22 44	28 28	40 40	48 48	81 81	92 92	116 116	503	1	503
5	1							0 0	1 1	4 4	12 12	15 15	16 16	21 42	27 27	39 39	47 47	80 80	91 91	115 115	489	1	489
6	1								0 0	3 3	11 11	14 14	15 15	20 40	26 26	38 38	46 46	79 79	90 90	114 114	476	1	476
9	1									0 0	8 8	11 11	12 12	17 34	23 23	35 35	43 43	76 76	87 87	111 111	440	1	440
17	1										0 0	3 3	4 4	9 18	15 15	27 27	35 35	68 68	79 79	103 103	352	1	352
20	1											0 0	1 1	6 12	12 12	24 24	32 32	65 65	76 76	100 100	322	1	322
21	1												0 0	5 10	11 11	23 23	31 31	64 64	75 75	99 99	313	1	313
26	2													0 0	6 6	18 18	26 26	59 59	70 70	94 94	273	2	546
32	1														0 0	12 12	20 20	53 53	64 64	88 88	237	1	237
44	1															0 0	8 8	41 41	52 52	76 76	177	1	177
52	1																0 0	33 33	44 44	68 68	145	1	145
85	1																	0 0	11 11	35 35	46	1	46
96	1																		0 0	24 24	24	1	24
120	1																			0 0	0 0	1	0
																						32	13560

Str.1 - Day time

$\hat{d}_1 = 27.65$ mm

xj	fj	xi:	0	1	2	3	4	8	12	14	18	19	22	26	30	32	39	40	50	53	61	90	102	105	110	120	165	178	308	Tj	fj	Tj fj
		fi:	5	1	1	4	1	1	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	2	1	1	1	1	1			
0	5		0 0	1 1	2 2	3 12	4 4	8 8	12 12	14 14	18 18	19 19	22 22	26 26	30 30	32 64	39 39	40 40	50 50	53 53	61 61	90 90	102 102	105 210	110 110	120 120	165 165	178 178	308 308	1758	5	8790
1	1			0 0	1 1	2 8	3 3	7 7	11 11	13 13	17 17	18 18	21 21	25 25	29 29	31 62	38 38	39 39	49 49	52 52	60 60	89 89	101 101	104 208	109 109	119 119	164 164	177 177	307 307	1727	1	1727
2	1				0 0	1 4	2 2	6 6	10 10	12 12	16 16	17 17	20 20	24 24	28 28	30 60	37 37	38 38	48 48	51 51	59 59	88 88	100 100	103 206	108 108	118 118	163 163	176 176	306 306	1697	1	1697
3	4					0 0	1 1	5 5	9 9	11 11	15 15	16 16	19 19	23 23	27 27	29 58	36 36	37 37	47 47	50 50	58 58	87 87	99 99	102 204	107 107	117 117	162 162	175 175	305 305	1668	4	6672
4	1						0 0	4 4	8 8	10 10	14 14	15 15	18 18	22 22	26 26	28 56	35 35	36 36	46 46	49 49	57 57	86 86	98 98	101 202	106 106	116 116	161 161	174 174	304 304	1643	1	1643
8	1							0 0	4 4	6 6	10 10	11 11	14 14	18 18	22 22	24 48	31 31	32 32	42 42	45 45	53 53	82 82	94 94	97 194	102 102	112 112	157 157	170 170	300 300	1547	1	1547
12	1								0 0	2 2	6 6	7 7	10 10	14 14	18 18	20 40	27 27	28 28	38 38	41 41	49 49	78 78	90 90	93 186	98 98	108 108	153 153	166 166	296 296	1455	1	1455
14	1									0 0	4 4	5 5	8 8	12 12	16 16	18 36	25 25	26 26	36 36	39 39	47 47	76 76	88 88	91 182	96 96	106 106	151 151	164 164	294 294	1414	1	1414
18	1										0 0	1 1	4 4	8 8	12 12	14 28	21 21	22 22	32 32	35 35	43 43	72 72	84 84	87 174	92 92	102 102	147 147	160 160	290 290	1327	1	1327
19	1											0 0	3 3	7 7	11 11	13 26	20 20	21 21	31 31	34 34	42 42	71 71	83 83	86 172	91 91	101 101	146 146	159 159	289 289	1307	1	1307
22	1												0 0	4 4	8 8	10 20	17 17	18 18	28 28	31 31	39 39	68 68	80 80	83 166	88 88	98 98	143 143	156 156	286 286	1250	1	1250
26	1													0 0	4 4	6 12	13 13	14 14	24 24	27 27	35 35	64 64	76 76	79 158	84 84	94 94	139 139	152 152	282 282	1178	1	1178
30	1														0 0	2 4	9 9	10 10	20 20	23 23	31 31	60 60	72 72	75 150	80 80	90 90	135 135	148 148	278 278	1110	1	1110
32	2															0 0	7 7	8 8	18 18	21 21	29 29	58 58	70 70	73 146	78 78	88 88	133 133	146 146	276 276	1078	2	2156
39	1																0 0	1 1	11 11	14 14	22 22	51 51	63 63	66 132	71 71	81 81	126 126	139 139	269 269	988	1	988
40	1																	0 0	10 10	13 13	21 21	50 50	62 62	65 130	70 70	80 80	125 125	138 138	268 268	967	1	967
50	1																		0 0	3 3	11 11	40 40	52 52	55 110	60 60	70 70	115 115	128 128	258 258	847	1	847
53	1																			0 0	8 8	37 37	49 49	52 104	57 57	67 67	112 112	125 125	255 255	814	1	814
61	1																				0 0	29 29	41 41	44 88	49 49	59 59	104 104	117 117	247 247	734	1	734
90	1																					0 0	12 12	15 30	20 20	30 30	75 75	88 88	218 218	473	1	473
102	1																						0 0	3 6	8 8	18 18	63 63	76 76	206 206	377	1	377
105	2																							0 0	5 5	15 15	60 60	73 73	203 203	356	2	712
110	1																								0 0	10 10	55 55	68 68	198 198	331	1	331
120	1																									0 0	45 45	58 58	188 188	291	1	291
165	1																										0 0	13 13	143 143	156	1	156
178	1																											0 0	130 130	130	1	130
308	1																												0 0	0	1	0
																															36	40085

Str.1 - Night time

$\hat{d}_1 = 61.86$ mm

	xi:	0	1	2	3	4	5	6	7	8	11	12	13	14	17	22	24	26	31	32	36	42	45	46	50	54	56	105	125	133	152	170	210	360			
xj	fj / fi:	17	14	4	5	4	2	1	1	3	1	3	2	1	2	1	1	1	1	2	1	2	1	1	2	1	2	1	1	1	1	1	1	1	Tj	fj	Tj fj
0	17	0/0	1/14	2/8	3/15	4/16	5/10	6/6	7/7	8/24	11/11	12/36	13/26	14/14	17/34	22/22	24/24	26/26	31/31	32/64	36/36	42/84	45/45	46/46	50/100	54/54	56/112	105/105	125/125	133/133	152/152	170/170	210/210	360/360	2120	17	36040
1	14		0/0	1/4	2/10	3/12	4/8	5/5	6/6	7/21	10/10	11/33	12/24	13/13	16/32	21/21	23/23	25/25	30/30	31/62	35/35	41/82	44/44	45/45	49/98	53/53	55/110	104/104	124/124	132/132	151/151	169/169	209/209	359/359	2054	14	28756
2	4			0/0	1/5	2/8	3/6	4/4	5/5	6/18	9/9	10/30	11/22	12/12	15/30	20/20	22/22	24/24	29/29	30/60	34/34	40/80	43/43	44/44	48/96	52/52	54/108	103/103	123/123	131/131	150/150	168/168	208/208	358/358	2002	4	8008
3	5				0/0	1/4	2/4	3/3	4/4	5/15	8/8	9/27	10/20	11/11	14/28	19/19	21/21	23/23	28/28	29/58	33/33	39/78	42/42	43/43	47/94	51/51	53/106	102/102	122/122	130/130	149/149	167/167	207/207	357/357	1954	5	9770
4	4					0/0	1/2	2/2	3/3	4/12	7/7	8/24	9/18	10/10	13/26	18/18	20/20	22/22	27/27	28/56	32/32	38/76	41/41	42/42	46/92	50/50	52/104	101/101	121/121	129/129	148/148	166/166	206/206	356/356	1911	4	7644
5	2						0/0	1/1	2/2	3/9	6/6	7/21	8/16	9/9	12/24	17/17	19/19	21/21	26/26	27/54	31/31	37/74	40/40	41/41	45/90	49/49	51/102	100/100	120/120	128/128	147/147	165/165	205/205	355/355	1872	2	3744
6	1							0/0	1/1	2/6	5/5	6/18	7/14	8/8	11/22	16/16	18/18	20/20	25/25	26/52	30/30	36/72	39/39	40/40	44/88	48/48	50/100	99/99	119/119	127/127	146/146	164/164	204/204	354/354	1835	1	1835
7	1								0/0	1/3	4/4	5/15	6/12	7/7	10/20	15/15	17/17	19/19	24/24	25/50	29/29	35/70	38/38	39/39	43/86	47/47	49/98	98/98	118/118	126/126	145/145	163/163	203/203	353/353	1799	1	1799
8	3									0/0	3/3	4/12	5/10	6/6	9/18	14/14	16/16	18/18	23/23	24/48	28/28	34/68	37/37	38/38	42/84	46/46	48/96	97/97	117/117	125/125	144/144	162/162	202/202	352/352	1764	3	5292
11	1										0/0	1/3	2/4	3/3	6/12	11/11	13/13	15/15	20/20	21/42	25/25	31/62	34/34	35/35	39/78	43/43	45/90	94/94	114/114	122/122	141/141	159/159	199/199	349/349	1668	1	1668
12	3											0/0	1/2	2/2	5/10	10/10	12/12	14/14	19/19	20/40	24/24	30/60	33/33	34/34	38/76	42/42	44/88	93/93	113/113	121/121	140/140	158/158	198/198	348/348	1637	3	4911
13	2												0/0	1/1	4/8	9/9	11/11	13/13	18/18	19/38	23/23	29/58	32/32	33/33	37/74	41/41	43/86	92/92	112/112	120/120	139/139	157/157	197/197	347/347	1609	2	3218
14	1													0/0	3/6	8/8	10/10	12/12	17/17	18/36	22/22	28/56	31/31	32/32	36/72	40/40	42/84	91/91	111/111	119/119	138/138	156/156	196/196	346/346	1583	1	1583
17	2														0/0	5/5	7/7	9/9	14/14	15/30	19/19	25/50	28/28	29/29	33/66	37/37	39/78	88/88	108/108	116/116	135/135	153/153	193/193	343/343	1508	2	3016
22	1															0/0	2/2	4/4	9/9	10/20	14/14	20/40	23/23	24/24	28/56	32/32	34/68	83/83	103/103	111/111	130/130	148/148	188/188	338/338	1393	1	1393
24	1																0/0	2/2	7/7	8/16	12/12	18/36	21/21	22/22	26/52	30/30	32/64	81/81	101/101	109/109	128/128	146/146	186/186	336/336	1349	1	1349
26	1																	0/0	5/5	6/12	10/10	16/32	19/19	20/20	24/48	28/28	30/60	79/79	99/99	107/107	126/126	144/144	184/184	334/334	1307	1	1307
31	1																		0/0	1/2	5/5	11/22	14/14	15/15	19/38	23/23	25/50	74/74	94/94	102/102	121/121	139/139	179/179	329/329	1207	1	1207
32	2																			0/0	4/4	10/20	13/13	14/14	18/36	22/22	24/48	73/73	93/93	101/101	120/120	138/138	178/178	328/328	1188	2	2376
36	1																				0/0	6/12	9/9	10/10	14/28	18/18	20/40	69/69	89/89	97/97	116/116	134/134	174/174	324/324	1120	1	1120
42	2																					0/0	3/3	4/4	8/16	12/12	14/28	63/63	83/83	91/91	110/110	128/128	168/168	318/318	1024	2	2048
45	1																						0/0	1/1	5/10	9/9	11/22	60/60	80/80	88/88	107/107	125/125	165/165	315/315	982	1	982
46	1																							0/0	4/8	8/8	10/20	59/59	79/79	87/87	106/106	124/124	164/164	314/314	969	1	969
50	2																								0/0	4/4	6/12	55/55	75/75	83/83	102/102	120/120	160/160	310/310	921	2	1842
54	1																									0/0	2/4	51/51	71/71	79/79	98/98	116/116	156/156	306/306	881	1	881
56	2																										0/0	49/49	69/69	77/77	96/96	114/114	154/154	304/304	863	2	1726
05	1																											0/0	20/20	28/28	47/47	65/65	105/105	255/255	520	1	520
25	1																												0/0	8/8	27/27	45/45	85/85	235/235	400	1	400
33	1																													0/0	19/19	37/37	77/77	227/227	360	1	360
52	1																														0/0	18/18	58/58	208/208	284	1	284
70	1																															0/0	40/40	190/190	230	1	230
10	1																																0/0	150/150	150	1	150
60	1																																	0/0	0	1	0
																																				83	136428

Str.2 - Day time

$\hat{d}_1 = 39.61$ mm

xi:		0	2	4	6	7	10	11	12	16	20	28	30	36	38	42	48	52	55	71	72	94	105	304	310	366	450	470	880	1170	1980			
fi:		23	2	2	1	1	2	1	1	1	1	2	1	1	2	2	2	1	1	1	1	1	2	1	1	1	1	1	1	1	1			
xj	fj																															Tj	fj	Tj fj
0	23	0 0	2 4	4 8	6 6	7 7	10 20	11 11	12 12	16 16	20 20	28 56	30 30	36 36	38 76	42 84	48 96	52 52	55 55	71 71	72 72	94 94	105 210	304 304	310 310	366 366	450 450	470 470	880 880	1170 1170	1980 1980	6966	23	160218
2	2		0 0	2 4	4 4	5 5	8 16	9 9	10 10	14 14	18 18	26 52	28 28	34 34	36 72	40 80	46 92	50 50	53 53	69 69	70 70	92 92	103 206	302 302	308 308	364 364	448 448	468 468	878 878	1168 1168	1978 1978	6892	2	13784
4	2			0 0	2 2	3 3	6 12	7 7	8 8	12 12	16 16	24 48	26 26	32 32	34 68	38 76	44 88	48 48	51 51	67 67	68 68	90 90	101 202	300 300	306 306	362 362	446 446	466 466	876 876	1166 1166	1976 1976	6822	2	13644
6	1				0 0	1 1	4 8	5 5	6 6	10 10	14 14	22 44	24 24	30 30	32 64	36 72	42 84	46 46	49 49	65 65	66 66	88 88	99 198	298 298	304 304	360 360	444 444	464 464	874 874	1164 1164	1974 1974	6756	1	6756
7	1					0 0	3 6	4 4	5 5	9 9	13 13	21 42	23 23	29 29	31 62	35 70	41 82	45 45	48 48	64 64	65 65	87 87	98 196	297 297	303 303	359 359	443 443	463 463	873 873	1163 1163	1973 1973	6724	1	6724
10	2						0 0	1 1	2 2	6 6	10 10	18 36	20 20	26 26	28 56	32 64	38 76	42 42	45 45	61 61	62 62	84 84	95 190	294 294	300 300	356 356	440 440	460 460	870 870	1160 1160	1970 1970	6631	2	13262
11	1							0 0	1 1	5 5	9 9	17 34	19 19	25 25	27 54	31 62	37 74	41 41	44 44	60 60	61 61	83 83	94 188	293 293	299 299	355 355	439 439	459 459	869 869	1159 1159	1969 1969	6602	1	6602
12	1								0 0	4 4	8 8	16 32	18 18	24 24	26 52	30 60	36 72	40 40	43 43	59 59	60 60	82 82	93 186	292 292	298 298	354 354	438 438	458 458	868 868	1158 1158	1968 1968	6574	1	6574
16	1									0 0	4 4	12 24	14 14	20 20	22 44	26 52	32 64	36 36	39 39	55 55	56 56	78 78	89 178	288 288	294 294	350 350	434 434	454 454	864 864	1154 1154	1964 1964	6466	1	6466
20	1										0 0	8 16	10 10	16 16	18 36	22 44	28 56	32 32	35 35	51 51	52 52	74 74	85 170	284 284	290 290	346 346	430 430	450 450	860 860	1150 1150	1960 1960	6362	1	6362
28	2											0 0	2 2	8 8	10 20	14 28	20 40	24 24	27 27	43 43	44 44	66 66	77 154	276 276	282 282	338 338	422 422	442 442	852 852	1142 1142	1952 1952	6162	2	12324
30	1												0 0	6 6	8 16	12 24	18 36	22 22	25 25	41 41	42 42	64 64	75 150	274 274	280 280	336 336	420 420	440 440	850 850	1140 1140	1950 1950	6116	1	6116
36	1													0 0	2 4	6 12	12 24	16 16	19 19	35 35	36 36	58 58	69 138	268 268	274 274	330 330	414 414	434 434	844 844	1134 1134	1944 1944	5984	1	5984
38	2														0 0	4 8	10 20	14 14	17 17	33 33	34 34	56 56	67 134	266 266	272 272	328 328	412 412	432 432	842 842	1132 1132	1942 1942	5942	2	11884
42	2															0 0	6 12	10 10	13 13	29 29	30 30	52 52	63 126	262 262	268 268	324 324	408 408	428 428	838 838	1128 1128	1938 1938	5866	2	11732
48	2																0 0	4 4	7 7	23 23	24 24	46 46	57 114	256 256	262 262	318 318	402 402	422 422	832 832	1122 1122	1932 1932	5764	2	11528
52	1																	0 0	3 3	19 19	20 20	42 42	53 106	252 252	258 258	314 314	398 398	418 418	828 828	1118 1118	1928 1928	5704	1	5704
55	1																		0 0	16 16	17 17	39 39	50 100	249 249	255 255	311 311	395 395	415 415	825 825	1115 1115	1925 1925	5662	1	5662
71	1																			0 0	1 1	23 23	34 68	233 233	239 239	295 295	379 379	399 399	809 809	1099 1099	1909 1909	5454	1	5454
72	1																				0 0	22 22	33 66	232 232	238 238	294 294	378 378	398 398	808 808	1098 1098	1908 1908	5442	1	5442
94	1																					0 0	11 22	210 210	216 216	272 272	356 356	376 376	786 786	1076 1076	1886 1886	5200	1	5200
105	2																						0 0	199 199	205 205	261 261	345 345	365 365	775 775	1065 1065	1875 1875	5090	2	10180
304	1																							0 0	6 6	62 62	146 146	166 166	576 576	866 866	1676 1676	3498	1	3498
310	1																								0 0	56 56	140 140	160 160	570 570	860 860	1670 1670	3456	1	3456
366	1																									0 0	84 84	104 104	514 514	804 804	1614 1614	3120	1	3120
450	1																										0 0	20 20	430 430	720 720	1530 1530	2700	1	2700
470	1																											0 0	410 410	700 700	1510 1510	2620	1	2620
880	1																												0 0	290 290	1100 1100	1390	1	1390
1170	1																													0 0	810 810	810	1	810
1980	1																														0 0	0	1	0
																																	60	355196

Str.2 - Night time

$\hat{d}_1 = 197.33$ mm

xj	fj	xi:	0	1	2	4	6	7	8	10	11	12	16	19	21	23	41	45	52	62	70	Tj	fj	Tj fj
		fi:	15	5	3	1	2	2	1	1	1	4	1	1	1	1	1	1	1	1	1			
0	15		0/0	1/5	2/6	4/4	6/12	7/14	8/8	10/10	11/11	12/48	16/16	19/19	21/21	23/23	41/41	45/45	52/52	62/62	70/70	467	15	7005
1	5			0/0	1/3	3/3	5/10	6/12	7/7	9/9	10/10	11/44	15/15	18/18	20/20	22/22	40/40	44/44	51/51	61/61	69/69	438	5	2690
2	3				0/0	2/2	4/8	5/10	6/6	8/8	9/9	10/40	14/14	17/17	19/19	21/21	39/39	43/43	50/50	60/60	68/68	414	3	1242
4	1					0/0	2/4	3/6	4/4	6/6	7/7	8/32	12/12	15/15	17/17	19/19	37/37	41/41	48/48	58/58	66/66	372	1	372
6	2						0/0	1/2	2/2	4/4	5/5	6/24	10/10	13/13	15/15	17/17	35/35	39/39	46/46	56/56	64/64	332	2	664
7	2							0/0	1/1	3/3	4/4	5/20	9/9	12/12	14/14	16/16	34/34	38/38	45/45	55/55	63/63	314	2	628
8	1								0/0	2/2	3/3	4/16	8/8	11/11	13/13	15/15	33/33	37/37	44/44	54/54	62/62	298	1	298
10	1									0/0	1/1	2/8	6/6	9/9	11/11	13/13	31/31	35/35	42/42	52/52	60/60	268	1	268
11	1										0/0	1/4	5/5	8/8	10/10	12/12	30/30	34/34	41/41	51/51	59/59	254	1	254
12	4											0/0	4/4	7/2	9/9	11/11	29/29	33/33	40/40	50/50	58/58	241	4	964
16	1												0/0	3/3	5/5	7/7	25/25	29/29	36/36	46/46	54/54	205	1	205
19	1													0/0	2/2	4/4	22/22	26/26	33/33	43/43	51/51	181	1	181
21	1														0/0	2/2	20/20	24/24	31/31	41/41	49/49	167	1	167
23	1															0/0	18/18	22/22	29/29	39/39	47/47	155	1	155
41	1																0/0	4/4	11/11	21/21	29/29	65	1	65
45	1																	0/0	7/7	17/17	25/25	49	1	49
52	1																		0/0	10/10	18/18	28	1	28
62	1																			0/0	8/8	8	1	8
70	1																				0/0	0	1	0
																							44	15243

Str.3 - Day time

$\hat{d}_1 = 15.75$ mm

xj	fj	xi: 0	2	4	6	7	10	11	12	18	30	34	37	60	76	85	110	120	154	170	330	430	560	610	1800	Tj	fj	Tj fj
		fi: 6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1	1	1			
0	6	0 0	2 2	4 4	6 6	7 7	10 10	11 11	12 12	18 18	30 30	34 34	37 37	60 60	76 76	85 85	110 110	120 120	154 154	170 170	330 660	430 430	560 560	610 610	1800 1800	5006	6	30036
2	1		0 0	2 2	4 4	5 5	8 8	9 9	10 10	16 16	28 28	32 32	35 35	58 58	74 74	83 83	108 108	118 118	152 152	168 168	328 656	428 428	558 558	608 608	1798 1798	4958	1	4958
4	1			0 0	2 2	3 3	6 6	7 7	8 8	14 14	26 26	30 30	33 33	56 56	72 72	81 81	106 106	116 116	150 150	166 166	326 652	426 426	556 556	606 606	1796 1796	4912	1	4912
6	1				0 0	1 1	4 4	5 5	6 6	12 12	24 24	28 28	31 31	54 54	70 70	79 79	104 104	114 114	148 148	164 164	324 648	424 424	554 554	604 604	1794 1794	4868	1	4868
7	1					0 0	3 3	4 4	5 5	11 11	23 23	27 27	30 30	53 53	69 69	78 78	103 103	113 113	147 147	163 163	323 646	423 423	553 553	603 603	1793 1793	4847	1	4847
10	1						0 0	1 1	2 2	8 8	20 20	24 24	27 27	50 50	66 66	75 75	100 100	110 110	144 144	160 160	320 640	420 420	550 550	600 600	1790 1790	4787	1	4787
11	1							0 0	1 1	7 7	19 19	23 23	26 26	49 49	65 65	74 74	99 99	109 109	143 143	159 159	319 638	419 419	549 549	599 599	1789 1789	4768	1	4768
12	1								0 0	6 6	18 18	22 22	25 25	48 48	64 64	73 73	98 98	108 108	142 142	158 158	318 636	418 418	548 548	598 598	1788 1788	4750	1	4750
18	1									0 0	12 12	16 16	19 19	42 42	58 58	67 67	92 92	102 102	136 136	152 152	312 624	412 412	542 542	592 592	1782 1782	4648	1	4648
30	1										0 0	4 4	7 7	30 30	46 46	55 55	80 80	90 90	124 124	140 140	300 600	400 400	530 530	580 580	1770 1770	4456	1	4456
34	1											0 0	3 3	26 26	42 42	51 51	76 76	86 86	120 120	136 136	296 592	396 396	526 526	576 576	1766 1766	4396	1	4396
37	1												0 0	23 23	39 39	48 48	73 73	83 83	117 117	133 133	586 586	393 393	523 523	573 573	1763 1763	4354	1	4354
60	1													0 0	16 16	25 25	550 50	60 60	94 94	110 110	270 540	370 370	500 500	550 550	1740 1740	4055	1	4055
76	1														0 0	9 9	34 34	44 44	78 78	94 94	254 508	354 354	484 484	534 534	1724 1724	3863	1	3863
85	1															0 0	25 25	35 35	69 69	85 85	245 290	345 345	475 475	525 525	1715 1715	3764	1	3764
110	1																0 0	10 10	44 44	60 60	220 440	320 320	450 450	500 500	1690 1690	3514	1	3514
120	1																	0 0	34 34	50 50	210 420	310 310	440 440	490 490	1680 1680	3424	1	3424
154	1																		0 0	16 16	176 352	276 276	406 406	456 456	1646 1646	3152	1	3152
170	1																			0 0	160 320	260 260	390 390	440 440	1630 1630	3040	1	3040
330	2																				0 0	100 100	230 230	280 280	1470 1470	2080	2	4160
430	1																					0 0	130 130	180 180	1370 1370	1680	1	1680
560	1																						0 0	50 50	1240 1240	1290	1	1290
610	1																							0 0	1190 1190	1190	1	1190
1800	1																								0 0	0	1	0
																											30	114912

Str.3 - Night time

$\hat{d}_1 = 255.36$ mm

xj	fj	xi: 0	1	2	3	4	5	6	8	10	11	14	16	19	24	28	32	42	48	56	57	58	62	76	127	160	180	190	210	219	320	340	370	610	920	1360	Tj	fj	Tj fj
		fi: 14	4	4	2	3	1	3	1	2	1	1	1	1	2	1	2	1	1	1	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1			
0	14	0 0	1 4	2 8	3 6	4 12	5 5	6 18	8 8	10 20	11 22	14 14	16 16	19 19	24 48	28 28	32 64	42 42	48 48	56 56	57 57	58 58	62 62	76 76	127 127	160 160	180 180	190 380	210 210	219 219	320 320	340 340	370 370	610 610	920 920	1360 1360	5887	14	82418
1	4		0 0	1 4	2 4	3 9	4 4	5 15	7 7	9 18	10 20	13 13	15 15	18 18	23 46	27 27	31 62	41 41	47 47	55 55	56 56	57 57	61 61	75 75	126 126	159 159	179 179	189 378	209 209	218 218	319 319	339 339	369 369	609 609	919 919	1359 1359	5837	4	23348
2	4			0 0	1 2	2 6	3 3	4 12	6 6	8 16	9 18	12 12	14 14	17 17	22 44	26 26	30 60	40 40	46 46	54 54	55 55	56 56	60 60	74 74	125 125	158 158	178 178	188 376	208 208	217 217	318 318	338 338	368 368	608 608	918 918	1358 1358	5791	4	23164
3	2				0 0	1 3	2 2	3 9	5 5	7 14	8 16	11 11	13 13	16 16	21 42	25 25	29 58	39 39	45 45	53 53	54 54	55 55	59 59	73 73	124 124	157 157	177 177	187 374	207 207	216 216	317 317	337 337	367 367	607 607	917 917	1357 1357	5749	2	11498
4	3					0 0	1 1	2 6	4 4	6 12	7 14	10 10	12 12	15 15	20 40	24 24	28 56	38 38	44 44	52 52	53 53	54 54	58 58	72 72	123 123	156 156	176 176	186 372	206 206	215 215	316 316	336 336	366 366	606 606	916 916	1356 1356	5709	3	17127
5	1						0 0	1 3	3 3	5 10	6 12	9 9	11 11	14 14	19 38	23 23	27 54	37 37	43 43	51 51	52 52	53 53	57 57	71 71	122 122	155 155	175 175	185 370	205 205	214 214	315 315	335 335	365 365	605 605	915 915	1355 1355	5672	1	5672
6	3							0 0	2 2	4 8	5 10	8 8	10 10	13 13	18 36	22 22	26 52	36 36	42 42	50 50	51 51	52 52	56 56	70 70	121 121	154 154	174 174	184 368	204 204	213 213	314 314	334 334	364 364	604 604	914 914	1354 1354	5636	3	16908
8	1								0 0	2 4	3 6	6 6	8 8	11 11	16 32	20 20	24 48	34 34	40 40	48 48	49 49	50 50	54 54	68 68	119 119	152 152	172 172	182 364	202 202	211 211	312 312	332 332	362 362	602 602	912 912	1352 1352	5570	1	5570
10	2									0 0	1 2	4 4	6 6	9 9	14 28	18 18	22 44	32 32	38 38	46 46	47 47	48 48	52 52	66 66	117 117	150 150	170 170	180 360	200 200	209 209	310 310	330 330	360 360	600 600	910 910	1350 1350	5506	2	11012
11	2										0 0	3 3	5 5	8 8	13 26	17 17	21 42	31 31	37 37	45 45	46 46	47 47	51 51	65 65	116 116	149 149	169 169	179 358	199 199	208 208	309 309	329 329	359 359	599 599	909 909	1349 1349	5476	2	10952
14	1											0 0	2 2	5 5	10 20	14 14	18 36	28 28	34 34	42 42	43 43	44 44	48 48	62 62	113 113	146 146	166 166	176 352	196 196	205 205	306 306	326 326	356 356	596 596	906 906	1346 1346	5392	1	5392
16	1												0 0	3 3	8 16	12 12	16 32	26 26	32 32	40 40	41 41	42 42	46 46	60 60	111 111	144 144	164 164	174 348	194 194	203 203	304 304	324 324	354 354	594 594	904 904	1344 1344	5330	1	5330
19	1													0 0	5 10	9 9	13 26	23 23	29 29	37 37	38 38	39 39	43 43	57 57	108 108	141 141	161 161	171 342	191 191	200 200	301 301	321 321	351 351	591 591	901 901	1341 1341	5260	1	5260
24	2														0 0	4 4	8 16	18 18	24 24	32 32	33 33	34 34	38 38	52 52	103 103	136 136	156 156	166 332	186 186	195 195	296 296	316 316	346 346	586 586	896 896	1336 1336	5135	2	10270
28	1															0 0	4 8	14 14	20 20	28 28	29 29	30 30	34 34	48 48	99 99	132 132	152 152	162 324	182 182	191 191	292 292	312 312	342 342	582 582	892 892	1332 1332	5043	1	5043
32	2																0 0	10 10	16 16	24 24	25 25	26 26	30 30	44 44	95 95	128 128	148 148	158 316	178 178	187 187	288 288	308 308	338 338	578 578	888 888	1328 1328	4955	2	9910
42	1																	0 0	6 6	14 14	15 15	16 16	20 20	34 34	85 85	118 118	138 138	148 296	168 168	177 177	278 278	298 298	328 328	568 568	878 878	1318 1318	4755	1	4755
48	1																		0 0	8 8	9 9	10 10	14 14	28 28	79 79	112 112	132 132	142 284	162 162	171 171	272 272	292 292	322 322	562 562	872 872	1312 1312	4641	1	4641
56	1																			0 0	1 1	2 2	6 6	20 20	71 71	104 104	124 124	134 268	154 154	163 163	264 264	284 284	314 314	554 554	864 864	1304 1304	4497	1	4497
57	1																				0 0	1 1	5 5	19 19	70 70	103 103	123 123	133 266	153 153	162 162	263 263	283 283	313 313	553 553	863 863	1303 1303	4480	1	4480
58	1																					0 0	4 4	18 18	69 69	102 102	122 122	132 264	152 152	161 161	262 262	282 282	312 312	552 552	862 862	1302 1302	4464	1	4464
62	1																						0 0	14 14	65 65	98 98	118 118	128 256	148 148	157 157	258 258	278 278	308 308	548 548	858 858	1298 1298	4404	1	4404
76	1																							0 0	51 51	84 84	104 104	114 228	134 134	143 143	244 244	264 264	294 294	534 534	844 844	1284 1284	4200	1	4200
127	1																								0 0	33 33	53 53	63 126	83 83	92 92	193 193	213 213	243 243	483 483	793 793	1233 1233	3545	1	3545
160	1																									0 0	20 20	30 60	50 50	59 59	160 160	180 180	210 210	450 450	760 760	1200 1200	3149	1	3149
180	1																										0 0	10 20	30 30	39 39	140 140	160 160	190 190	430 430	740 740	1180 1180	2929	1	2929
190	2																											0 0	20 20	29 29	130 130	150 150	180 180	420 420	730 730	1170 1170	2829	2	5658
210	1																												0 0	9 9	110 110	130 130	160 160	400 400	710 710	1150 1150	2669	1	2669
219	1																													0 0	101 101	121 121	151 151	391 391	701 701	1141 1141	2606	1	2606
320	1																														0 0	20 20	50 50	290 290	600 600	1040 1040	2000	1	2000
340	1																															0 0	30 30	270 270	580 580	1020 1020	1900	1	1900
370	1																																0 0	240 240	550 550	990 990	1780	1	1780
610	1																																	0 0	310 310	750 750	1060	1	1060
920	1																																		0 0	440 440	440	1	440
1360	1																																			0 0	0	1	0
																																						64	308067

Str.4 - Day time

$\hat{d}_1 = 150.42$ mm

NO: 11197

xj	fj	xi: 0	1	3	4	12	20	21	26	40	42	55	65	70	105	110	120	125	126	130	148	180	230	240	345	390	410	420	770	1950	6600	Tj	fj	Tj fj
		fi: 12	1	2	2	3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1			
0	12	0 0	1 1	3 6	4 8	12 36	20 20	21 21	26 26	40 40	42 42	55 55	65 65	70 70	105 105	110 110	120 120	125 125	126 126	130 130	148 148	180 180	230 230	240 240	345 345	390 390	410 410	420 420	770 770	1950 1950	6600 6600	12789	12	153468
1	1		0 0	2 4	3 6	11 33	19 19	20 20	25 25	39 39	41 41	54 54	64 64	69 69	104 104	109 109	119 119	124 124	125 125	129 129	147 147	179 179	229 229	239 239	344 344	389 389	409 409	419 419	769 769	1949 1949	6599 6599	12956	1	12956
3	2			0 0	1 2	9 27	17 17	18 18	23 23	37 37	39 39	52 52	62 62	67 67	102 102	107 107	117 117	122 122	123 123	127 127	145 145	177 177	227 227	237 237	342 342	387 387	407 407	417 417	767 767	1947 1947	6597 6597	12692	2	25384
4	2				0 0	8 24	16 16	17 17	22 22	36 36	38 38	51 51	61 61	66 66	101 101	106 106	116 116	121 121	122 122	126 126	144 144	176 176	226 226	236 236	341 341	386 386	406 406	416 416	766 766	1946 1946	6596 6596	12662	2	25324
12	3					0 0	8 8	9 9	14 14	28 28	30 30	43 43	53 53	58 58	93 93	98 98	108 108	113 113	114 114	118 118	136 136	168 168	218 218	228 228	333 333	378 378	398 398	408 408	758 758	1938 1938	6588 6588	12438	3	37314
20	1						0 0	1 1	6 6	20 20	22 22	35 35	45 45	50 50	85 85	90 90	100 100	105 105	106 106	110 110	128 128	160 160	210 210	220 220	325 325	370 370	390 390	400 400	750 750	1930 1930	6580 6580	12230	1	12230
21	1							0 0	5 5	19 19	21 21	34 34	44 44	49 49	84 84	89 89	99 99	104 104	105 105	109 109	127 127	159 159	209 209	219 219	324 324	369 369	389 389	399 399	749 749	1929 1929	6579 6579	12214	1	12214
26	1								0 0	14 14	16 16	29 29	39 39	44 44	79 79	84 84	94 94	99 99	100 100	104 104	122 122	154 154	204 204	214 214	319 319	364 364	384 384	394 394	744 744	1924 1924	6574 6574	12099	1	12099
40	1									0 0	2 2	15 15	25 25	30 30	65 65	70 70	80 80	85 85	86 86	90 90	108 108	140 140	190 190	200 200	305 305	350 350	370 370	380 380	730 730	1910 1910	6560 6560	11791	1	11791
42	1										0 0	13 13	23 23	28 28	63 63	68 68	78 78	83 83	84 84	88 88	106 106	138 138	188 188	198 198	303 303	348 348	368 368	378 378	728 728	1908 1908	6558 6558	11749	1	11749
55	1											0 0	10 10	15 15	50 50	55 55	65 65	70 70	71 71	75 75	93 93	125 125	175 175	185 185	290 290	335 335	355 355	365 365	715 715	1895 1895	6545 6545	11489	1	11489
65	1												0 0	5 5	40 40	45 45	55 55	60 60	61 61	65 65	83 83	115 115	165 165	175 175	280 280	325 325	345 345	355 355	705 705	1885 1885	6535 6535	11299	1	11299
70	1													0 0	35 35	40 40	50 50	55 55	56 56	60 60	78 78	110 110	160 160	170 170	275 275	320 320	340 340	350 350	700 700	1880 1880	6530 6530	11209	1	11209
105	1														0 0	5 5	15 15	20 20	21 21	25 25	43 43	75 75	125 125	135 135	240 240	285 285	305 305	315 315	665 665	1845 1845	6495 6495	10614	1	10614
110	1															0 0	10 10	15 15	16 16	20 20	38 38	70 70	120 120	130 130	235 235	280 280	300 300	310 310	660 660	1840 1840	6490 6490	10534	1	10534
120	1																0 0	5 5	6 6	10 10	28 28	60 60	110 110	120 120	225 225	270 270	290 290	300 300	650 650	1830 1830	6480 6480	10304	1	10304
125	1																	0 0	1 1	5 5	23 23	55 55	105 105	115 115	220 220	265 265	285 285	295 295	645 645	1825 1825	6475 6475	10314	1	10314
126	1																		0 0	4 4	22 22	54 54	104 104	114 114	219 219	264 264	284 284	294 294	644 644	1824 1824	6474 6474	10301	1	10301
130	1																			0 0	18 18	50 50	100 100	110 110	215 215	260 260	280 280	290 290	640 640	1820 1820	6470 6470	10253	1	10253
148	1																				0 0	32 32	82 82	92 92	197 197	242 242	262 262	272 272	622 622	1802 1802	6452 6452	10055	1	10055
180	1																					0 0	50 50	60 60	165 165	210 210	230 230	240 240	590 590	1770 1770	6420 6420	9735	1	9735
230	1																						0 0	10 10	115 115	160 160	180 180	190 190	540 540	1720 1720	6370 6370	9205	1	9205
240	1																							0 0	105 105	150 150	170 170	180 180	530 530	1710 1710	6360 6360	9205	1	9205
345	1																								0 0	45 45	65 65	75 75	425 425	1605 1605	6255 6255	8470	1	8470
390	1																									0 0	20 20	30 30	380 380	1560 1560	6210 6210	8200	1	8200
410	1																										0 0	10 10	360 360	1540 1540	6190 6190	8100	1	8100
420	1																											0 0	350 350	1530 1530	6180 6180	8060	1	8060
770	1																												0 0	1180 1180	5830 5830	7010	1	7010
1950	1																													0 0	4650 4650	4650	1	4650
6600	1																														0 0	0	1	0
																																	45	493504

Str.4 - Night time

$\hat{d}_1 = 487.41$ mm